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Abstract

Variable time lags are a possible source of randomness in relationships between economic time series. They are modelled here by means of variable regression coefficients. The model entails heteroscedastic residuals with a negative serial correlation and can be estimated by the Kalman filter. This extension of the traditional regression model is highly significant for the relationship between quarterly values of wages and prices in Iceland.

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I. Introduction

Irregular time lags are a plausible source of randomness in causal relationships. But in most econometric models the stochastic element is introduced by additive residuals, representing measurement errors or disturbances, independent of the regressors. A common explanation of errors in predictions of the inflation rate in Iceland has been that the expected effect of a wage increase or devaluation was delayed or came too soon. We introduce models allowing for variability of this kind. Other econometric techniques applied here are described in textbooks, e.g. by Hendry (1995). In spite of great changes in the magnitude of the inflation, parameters in models describing inflation by means of changes in wages and import prices appear fairly stable. However, estimation with variable time lags reveals heteroscedasticity associated with lagged wage changes. We compare different possibilities of modelling this effect.

Inflation was higher in Iceland than in most Western European countries from the Second World War until 1990. There are difficulties in obtaining long records of comparable wage observations and the present analysis only goes back to 1962. During the first years of this period the annual inflation rate was of the order of 10 per cent. Wage indexation was fixed by law at the end of 1964, but had no immediate noticeable effect upon inflation. In 1973 -1974 inflation increased considerably which coincides with less reluctance to devalue the currency and formal abolition of fixed exchange rates at the end of 1973. Annual inflation from 1974 to 1983 was on average about 50 per cent. In 1983 wage indexation was terminated and various other arrangements made to combat inflation and it decreased to an annual average of about 20 per cent. A wage agreement in 1990, including most of the labour force, was intended to bring inflation down to normal values in industrialised countries and since then the annual rate has been about 3 per cent. Andersen and Guðmundsson (1998) provide a more detailed description of the inflation in Iceland in the post war period. Pétursson (1998) presents a recent study of Icelandic inflation.

With high and variable inflation there has been much demand for inflation predictions and various institutions have produced them from time to time. They have mainly been based on weighted averages of past and present changes in wages and import prices and past values of the inflation itself. The more sophisticated versions have included error correction terms and time series modelling of residuals.

The best equations have an R^2 of almost 0.9 and standard error of residuals of logarithmic values about 0.014. In periods of high and variable inflation, forecasts based on such equations are valuable, provided the independent variables can be predicted with

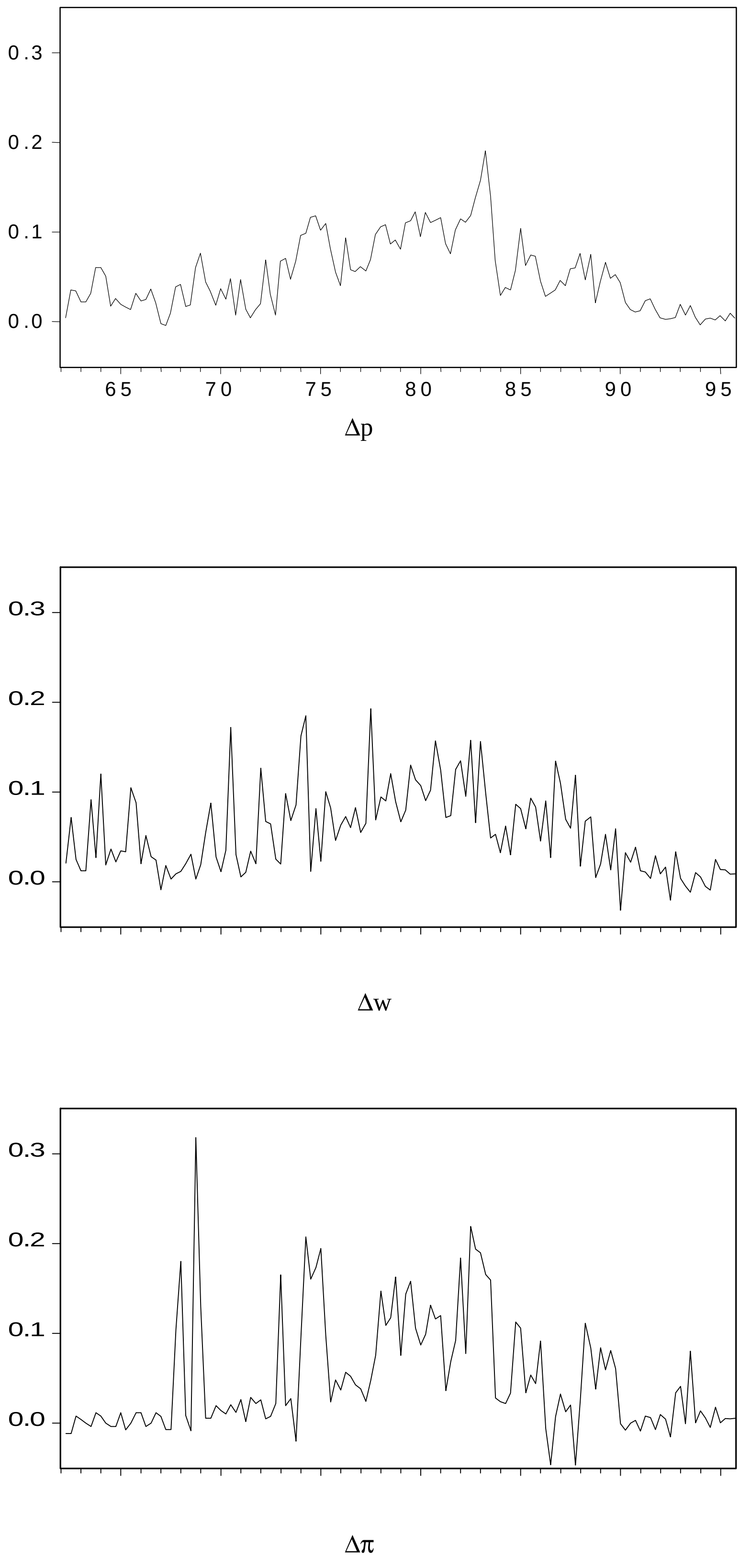

Figure 1. Graphs of first differences of quarterly logarithmic values of the consumer price index, wage index and import price index.

satisfactory accuracy. Most of the time the wages were easily predictable. Although the formal power of wage settlements is vested in a large number of trade unions, in practice they have been highly centralised. A rational wage forecaster therefore knew the future values of wages up to the next general settlement fairly well or could calculate them in a model based on the rules for the indexation (which was calculated from the price index with three months time lag). Wage contracts were usually for periods of one or two years.

No comparable information about import prices was available to forecasters. Most of the variation was connected with devaluations. Prediction by econometric models was obviously pointless in the first years when the exchange rate was kept constant for long periods and devaluations were very large. But even after 1973 no model, based on past values, explains a large proportion of exchange rate variations. I suppose that we can describe the official exchange rate policy for most of the period examined here something like this: As the authorities were anxious to combat inflation they were reluctant to devalue. However, the exchange rate had to be adjusted so that fishing and fish processing, which produce most of the country's exports, remained profitable. The adjustment was much slower in the first years than after 1973 and exchange rates may again have become less flexible in the later years, although certainly not as rigid as in the sixties.

II. The time series

The present analysis will be concerned with 3 series of quarterly averages.

p = consumer price index

w = wage index

π = price index of imports

Log-values of all series are used. First differences of the series are presented in Figure 1.

Subsidies were eliminated from the consumer price index. The wage index covers manual workers, including craftsmen. Measurement of a price index of imports to Iceland is a difficult task because of large irregular and seasonal variations in the quantities and kinds of imports. Results from attempts to estimate an import price index by traditional methods have not looked credible. The present index is calculated in a different way. The IMF price index of exports from industrialised countries, adjusted for the exchange rate, represents all imports except oil products.

Pétursson (1998) carried out a formal analysis of the time series properties of the present series up to 1993 and concluded that they could be regarded as I(1).

It is difficult to obtain reliable estimates of cointegration relationships with the present series. They are all broad indices and systematic errors in the measurement of long term changes might be substantial. In particular, there has been a great change in working hours and the composition of the labour force. Unit labour costs are a more appropriate variable than a wage index for explaining changes in consumer prices, but attempts to use available data on productivity have not improved the goodness of fit. One reason for this may be that changes in catches and prices of fish have great effect on income in all sectors of the economy. There is also strong evidence, both economic and statistic, that production and export values ought to be included in relationships for wages and exchange rates.

Another reason why prospects of obtaining reliable estimates of cointegration relationships from the present data seem poor is that, regardless of the outcome of Dickey-Fuller tests, first differences of the wage- and price series are not credible representations of stationary processes. The mean values and autocovariance functions of such processes are independent of time. An old fashioned check on whether it is practical to regard a series of observed values as stationary is whether the correlogram is almost zero at lags bigger than some value, considerably shorter than the length of the series. This is not the case here. In fact not even the sample means are useful estimates of mean values of inflation or wage increases. An ARIMA(p,1,q) model predicts that the inflation will fairly rapidly approach the mean value, which amounts to about 22 per cent annually. After enjoying a spell of over 6 years of stability with inflation about 3 per cent per year, few economists would now support this prediction. The assumption that inflation is an I(2) process does not solve the problem; according to an ARIMA(p,2,q) model we might well be experiencing substantial deflation in a year or two.

It is more appropriate to regard the inflation as an evolutionary process or a process with long memory. But with only 136 quarterly observations there is little scope for time series analysis based on these concepts. The present analysis is based on first differences and error correction terms.

Two trend variables are included as substitutes for productivity changes and systematic measurement errors:

T1 =1,2,... from 1962:1 to 1995:4

T2= 0 from 1962:1 to 1973:4 and 1,2,... from 1974:1 to 1995:4.

III. Variable time lags

As mentioned in the introduction, a common explanation of erroneous predictions of price changes in Iceland has been that the effects of a wage increase (or devaluation) came too soon or were delayed. To some extent the error correction term $(w\text{-}p)_{t-1}$ deals with effects of this kind. If an expected increase in p due to a wage increase has been delayed, the error correction term increases and produces bigger values of Δp until equilibrium is reached. However, the function of $(w\text{-}p)_{t-1}$ in a model of inflation is to preserve the long-term equilibrium rather than account for irregular time lags. With a coefficient of about 0.1 for the error correction term, adjustment is a matter of years. We have experimented with a model where the whole adjustment takes place in the next quarter:

Let the coefficients of Δw be variable so that the price equation is

$$\Delta p_t = \sum_{j=0}^{k} \beta_{jt}\Delta w_{t-j} + \text{other terms} + \varepsilon_t. \qquad (1)$$

Each β_{jt} has a fixed mean b_j. The first coefficient is

$$\beta_{0t} = b_0 + \delta_{0t}$$

where b_0 is a constant value and δ_{0t} represents the irregular timing of the effect of Δw_t. The coefficient of Δw_{t-1} has corresponding terms, but it also adjusts for the deviation of $\beta_{0,t-1}$ from b_0. The same applies to the following coefficients so that

$$\beta_{jt} = b_j - \delta_{j-1,t-1} + \delta_{jt}, \qquad 0 < j < k.$$

The last coefficient only adjusts for the irregularity introduced by $\beta_{k-1,t-1}$:

$$\beta_{kt} = b_k - \delta_{k-1,t-1}.$$

All δ_{it} are $N(0;\omega_i^2)$,

$$E\{\delta_{it}\delta_{js}\} = 0 \quad \forall\ t \neq s$$

and

$$E\{\delta_{it}\delta_{jt}\} = \rho_{ij}\omega_i\omega_j \quad \forall\ i \neq j$$

where ρ_{ij} are correlation coefficients.

The values of β_{jt} are time series. Let us eliminate the lagged residuals and write β_{jt} as autoregressive series. For k = 3 the equations are

$$\beta_{1t} = b_1 - [\beta_{0,t-1} - b_0] + \delta_{1t},$$

$$\beta_{2t} = b_2 - [\beta_{1,t-1} - b_1 + \beta_{0,t-2} - b_0] + \delta_{t2},$$

$$\beta_{3t} = b_3 - [\beta_{2,t-1} - b_2 + \beta_{1,t-2} - b_1 + \beta_{0,t-3} - b_0] + \delta_{t3}.$$

We define three more elements for the vector $\boldsymbol{\beta_t}$:

$$\beta_{4t} = \beta_{0,t-1},$$

$$\beta_{5t} = \beta_{1,t-1},$$

$$\beta_{6t} = \beta_{0,t-2}$$

and write

$$\boldsymbol{\beta_t} = \phi\boldsymbol{\beta_{t-1}} + \mathbf{B} + \boldsymbol{\delta_t}, \qquad (2)$$

where

$$\mathbf{B} = (b_0,b_0+b_1,b_0+b_1+b_2,b_0+b_1+b_2+b_3,0,0,0)',$$

$$\boldsymbol{\delta_t} = (\delta_{0t},\delta_{1t},\delta_{2t},0,0,0,0)'$$

and the elements of ϕ are obtained from the autoregressive forms of β_{jt}.

Pagan (1980) and Harvey and Phillips (1982) discuss estimation of regression coefficients following an autoregressive model. Here the unobserved series $\boldsymbol{\beta_t}$ and the parameters of equation (1), including b_i, ω_i^2 and ρ_{ij}, are estimated by the Kalman filter and the likelihood function of the one step prediction errors as described by Harvey (1989).

In practice the number of parameters that can be estimated is limited. A strong simplifying assumption has already been made by excluding variations in timing by more than one interval. This can be achieved by working with an appropriate time interval between observations. Economic conditions producing a delay in the effects of one Δw_{t-j} would usually have similar effects on other lagged or present values of this variable at the same time so that positive and rather large values of ρ_{ij} would be the rule if variable time lags of this kind are present. A convenient simplification is to let all ρ_{ij} take the same value. Another possibility for simplification is to link the magnitude of the time-variations with respective average effect. By using this and assuming that the time-variations are fully co-ordinated they can be modelled with one additional parameter:

$$\Delta p_t = (1+\delta_t) \sum_{j=0}^{k} b_j \Delta w_{t-j} - \delta_{t-1} \sum_{j=0}^{k} b_j \Delta w_{t-1-j} + \text{other terms} + \varepsilon_t. \qquad (3)$$

where $\delta_t \sim IN(0;\omega^2)$.

Our model of variable time lags corresponds to an MA(1) relationship with a negative coefficient of the lagged residual. Both residual variance and the coefficient of the lagged term depend upon Δw_t and it's lagged values. Modelling of relationships where variable time lags are present without taking them into account should therefore produce heteroscedasticity and a negative MA(1) term.

IV. Results

Fixed coefficients

The parameters of the equation

$$\Delta p_t = c + \alpha \Delta p_{t-1} + \sum_{0}^{1} \beta_j \Delta w_{t-j} + \sum_{0}^{1} \gamma_j \Delta \pi_{t-j} + \theta_1 (w-p)_{t-1} + \theta_2 (\pi-p)_{t-1}$$

$$+ \theta_3 T1 + \theta_4 T2 + \varepsilon_t + \theta_5 \varepsilon_{t-1} \qquad (4)$$

are presented in Table 1. The following specification tests were carried out:

Normality: $\chi^2(2) = 3.23$ $(p=0.199)$

Autocorrelation: $F(2,119) = 2.21$ $(p=0.114)$

ARCH: $F(1,131) = 0.074$ $(p=0.787)$

Heteroscedasticity: $F(18,113) = 1.49$ $(p=0.104)$

Heteroscedasticity: $F(53,78) = 1.96$ $(p = 0.0041)$

Ramsey's reset test: $F(1,121) = 3.66$ $(p = 0.058)$

Other lags in the tests for autocorrelation or ARCH give bigger p-values. The homogeneity restriction on the parameters of p_{t-1}, w_{t-1} and π_{t-1} was tested and accepted (p = 0.540), indicating that the trend terms are adequate representatives of missing variables and systematic measurement errors in the long-term behaviour.

Table 1

Estimation results. Standard deviations of respective parameters in parentheses

parameters	equation (4)	equation (1)	equation (3)
Δp_{t-1}	0.323	0.306	0.266
	(0.100)	(0.069)	(0.072)
Δw_t	0.164	0.146	0.121
	(0.032)	(0.029)	(0.032)
Δw_{t-1}	0.038	0.079	0.111
	(0.039)	(0.038)	(0.030)
$\Delta \pi_t$	0.193	0.182	0.186
	(0.025)	(0.018)	(0.018)
$\Delta \pi_{t-1}$	0.037	0.051	0.058
	(0.033)	(0.023)	(0.021)
$p_{t-1} - w_{t-1}$	-0.115	-0.111	-0.114
	(0.034)	(0.025)	(0.026)
$p_{t-1} - \pi_{t-1}$	-0.076	-0.073	-0.078
	(0.024)	(0.017)	(0.018)
T1	-0.00128	-0.00122	-0.00124
	(0.00036)	(0.00028)	(0.00029)
T2	0.00108	0.00103	0.00105
	(0.00031)	(0.00025)	(0.00027)
c	0.093	0.085	0.089
	(0.028)	(0.019)	(0.020)
ma(1)	-0.194	0.59	0.75
	(0.179)	(0.33)	(0.34)
ω_0		0.036	
		(0.021)	
ω_1		0.087	
		(0.029)	
ω			0.54
			(0.20)
ρ_{12}		1.0	1.0
log(L)	386.77	399.44	397.44
σ	0.0129	0.0065	0.0063
rmse.	0.0129	0.0131	0.0132

In view of changing economic conditions, described in the introduction, it is appropriate to examine stability of the relationship with respect to 4 periods:

I 1963:1 - 1973:4
II 1974:1 - 1983:3
III 1983:4 - 1990:2
IV 1990:3 - 1995:4

Estimation with joint parameters except Var(ε) = σ_i^2, where i = I, II, III and IV for respective period, gave log(L) = 396.88 which is a highly significant increase. The estimated values were:

i	I	II	III	IV
$\hat{\sigma}_i$	0.0136	0.0140	0.0150	0.0053
$\overline{\Delta p}$	0.032	0.100	0.051	0.008

The relevance of the MA-term in equation (4) is questionable according to the results in Table 1 and there is no indication of serial correlation when it is left out. But when it is included its parameter varies greatly in the four periods. A likelihood ratio test of the hypothesis that the remaining parameters are the same in all periods gives $\chi^2(26) = 37.04$, (p=0.074). Test of the hypothesis of constant parameters apart from Var(ε_t) when no MA term is included gives a $\chi^2(26) = 28.52$, (p = 0.333).

The most plausible explanation of variations in Var(ε_t) in equation (4) is that it varies with the inflation. However, it is obvious from comparison of $\hat{\sigma}_i$ with the average inflation in respective period, presented above, that this is an inadequate theory.

The first F-test for heteroscedasticity reported above is obtained from regression of $\hat{\varepsilon}_t^2$ on the independent variables in the equation and their squared values. It does not present a strong indication of systematic variations in Var(ε_t). Examination of the regression coefficients reveals that the coefficients of Δp and Δp^2 have $|$t-values$|$ smaller than one and the only coefficients with $|$t-values$|$ > 1 are those of $(w-p)_{t-1}$ and $(\pi-p)_{t-1}$.

The second test for heteroscedasticity is obtained by adding all products of two independent variables to the set of regressors in the first test. The test statistic looks highly significant. However, the number of regressors is very large compared with the number of observations so that this is not a reliable indicator of misspecification or heteroscedasticity. There are now 6 variables with |t-values|> 1, the biggest value is 2.8. These variables all include Δp_{t-1} and are Δp_{t-1}, Δp_{t-1}^2, $\Delta p_{t-1}(\pi_{t-1}-p_{t-1})$, $\Delta p_{t-1}(w_{t-1}-p_{t-1})$, $\Delta p_{t-1}T1$ and $\Delta p_{t-1}T2$.

Variable time lags

We examined the possibility of variable time lags in the effects of wages and import prices. There was no indication of this in connection with the import prices and only results with variable coefficients of Δw_{t-i} will be presented. Table 1 gives the results of estimating the parameters of equation (1) with β_{jt} according to equation (2), k=2 and b_2=0. The log-likelihood is 399.44, an increase by 12.67 by including non-zero variances of the residuals δ_{0t} and δ_{1t}.

Modelling of variable time lags of wage increases is overparametrized by separate estimation of ω_1, ω_2 and ρ_{12}. The likelihood function is not sensitive to the value of ρ_{12}; logL only decreases to 399.28 when ρ_{12}=0. However, in this modelling of variable time lags, zero is no more plausible choice of the value of correlations between the residuals than one. In the following comparisons of different model formulations the value will be fixed at 0.5, but the results would be very similar with either 0 or 1.

As pointed out previously, variable time lags are associated with heteroscedasticity and an MA(1) form of residual correlation. The indications of heteroscedasticity obtained when the model was estimated without variable time lags were not of the form predicted by equation (2), which would be

$$\mathrm{Var}\{\varepsilon_t\} = \sigma^2(1 + c_0\Delta w_t^2 + c_1\Delta w_{t-1}^2 + c_2\Delta w_{y-2}^2)$$

when no correlation between δ_{it} is assumed. Estimation of equation (4) with this formulation of $\mathrm{Var}\{\varepsilon_t\}$ gave logL = 398.37, but only c_1 was significantly different from 0. The estimated MA(1) term in equation (4) was small and not significant, but had the expected sign when variable timing is present but not modelled.

We have presented three extensions of equation (4): variations with time, represented by the parameters σ_i, variable time lags, represented by ω_i and variation of $\mathrm{Var}\{\varepsilon_t\}$ with $c_1\Delta w_{t-1}^2$. Each extension is separately highly significant according to the increase in logL and they all describe variations in $\mathrm{Var}\{\varepsilon_t\}$ with time. Let us now investigate to what extent these effects can be regarded as different manifestations of the same effect. Table 2 presents the values of logL for equation (4) extended by all sets of these parameters, all combinations of two of them and each of them separately.

We have calculated the likelihood ratio tests for three hypotheses:

H_0^1: $\sigma_I = \sigma_{II} = \sigma_{III} = \sigma_{IV}$,

H_0^2: $\omega_1 = \omega_2 = 0$,

H_0^3: $c_1 = 0$.

Table 2

Comparison of models of variations in residual variance

parameters added to eq.(4)	number of parameters	logL	hypotheses
σ_i, ω_i, c_1	18	403.51	
σ_i, ω_i,	17	402.34	H_0^3: p=0.126
σ_i, c_1	16	401.58	H_0^2: p=0.145
ω_i, c_1	15	400.42	H_0^1: p=0.103
σ_i	15	396.88	$H_0^2 \cap H_0^3$: p=0.004 H_0^2: p=0.004 H_0^3: p=0.002
ω_i	14	399.43	$H_0^1 \cap H_0^3$ p=0.085 H_0^1: p=0.121 H_0^3: p=0.159
c_1	13	397.53	$H_0^1 \cap H_0^2$ p=0.035 H_0^1: p=0.044 H_0^2: p=0.056

The first three rows below the full model present tests of each hypothesis against the alternative of using all these parameters freely. There is no strong evidence against any of the hypotheses so that any combination of two of the three extensions examined is acceptable.

We proceed to test the restriction to include only one extension. Each of them is tested against the alternative of including all three and also against including either of the other extensions. The simplification of using only different residual variance in the four specified periods is firmly rejected. There is little evidence against using only variable time lags but more against using only variation of Var{ε_t} with $c_1\Delta w_{t-1}^2$.

Apart from the goodness of fit indicated by the likelihood function, some weak additional support for variable time lags is provided by the sign of estimated MA(1) terms. On the other hand, the rmse. would be expected to decrease by estimating a correctly specified model of variable time lags, but in fact a small increase is observed.

Another specification of variable time lags with the correlations fixed at one and the variance proportional to the squared coefficient at respective lag was presented in equation

(3). Extension of equation (4) by this one-parameter model gives logL = 397.44 and the estimated parameters are presented in the last column of Table 1. The p-value of the likelihood ratio test of $H_0^1 \cap H_0^3$ is 0.051. The model obtained by only including the residuals δ_{1t} has logL = 397.90 and the p-value for $H_0^1 \cap H_0^3$ is 0.057.

Conclusions

In regression models with fixed coefficients the stochastic element is introduced in the form of additive errors, independent of the regressors. Variable time lags are commonly mentioned as a source of irregularities in economic relationships. They are modelled here by means of variable coefficients. The specification corresponds to residuals with negative serial correlations and the variance is connected to the magnitude of respective regressor variable.

This extension of the traditional model appears to be highly significant in the relationship between wages and prices in Iceland. However, it is not possible to distinguish with certainty whether the observed variability is in fact caused by variable time lags or some other changes in variability, associated with wage changes.

Variable time lags or residual variance are most strongly associated with wage changes, lagged by one quarter. This is remarkable in view of the fact that the regression coefficient of this term is small and barely significant. But even if it is included, the tests of heteroscedasticity, readily available in econometric software, provide no indication of such relationship.

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