CENTRAL BANK OF ICELAND

WORKING PAPER

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2024|96

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Central Bank of Iceland Working Papers are published by the Economics and Monetary Policy Department of the Central Bank of Iceland.

ISSN 1028-9445

TESTING OPTIMAL MONETARY POLICY IN A CURRENCY UNION[*]

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August 29, 2024

Abstract

This paper presents a framework for testing the optimality of monetary policy decisions made by a central bank in a monetary union. Applying the framework to test the European Central Bank's monetary policy decisions we find several instances of optimization failures in its use of the Forward Guidance and Quantitative Easing instruments. We cannot reject optimality in its use of the Target Rate instrument. We find signs of heterogeneity in the optimal prescriptions for the individual member countries with respect to the union level prescription. Additionally, we find many instances of optimization failure at the country level for all instruments. Assuming each country has a country specific version of the union loss function we provide a measure of the cost of abandoning independent monetary policy by joining a union. The results indicate that the price of Euro membership is higher for the peripheral economies.

JEL classification: C32, E31, E32, E52, E58, E61, E65.

Keywords: Monetary policy, Macroeconomic policy design, optimal policy, instrumental variable local projections.

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[*] I would like to thank Dante Amengual, Regis Barnichon, Christian Brownlees, Valeria Gargiulo, Petra Ivas, Adam Lee, Zidong Lin, Geert Mesters, Katerina Petrova, Barbara Rossi, Andre B.M. Souza, Stefán Thorarinsson, and participants at the 2020 Barcelona GSE PhD Jamboree, and the internal Central Bank of Iceland seminar for helpful comments, discussions, and suggestions.

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1 Introduction

How do we determine whether the monetary policy decisions of a central bank in a monetary union are optimal? Despite a large and vibrant debate, both in public and academic circles, about whether the decisions of a central bank are the correct ones at any given point in time, there are remarkably few ways in which to formally test whether a given monetary policy decision is optimal or not. This is juxtaposed to the plethora of papers studying optimal policy rules using structural macroeconomic models following the guideline of Lucas (1976).[1] These papers are important when it comes to the overarching design of the policy regime but they are less informative when it comes to individual policy decisions. In practice however, monetary policy decisions are not made according to fixed rules as specified by a single structural model. In this paper we are interested in testing the optimality of practical monetary policy decisions, i.e., decisions that do not rely on a specific structural macroeconomic model.

In the present paper, we address this issue and present a framework to answer the question of whether the monetary policy decisions of a central bank in a currency union are optimal or not. More precisely, we present a framework to test whether the central bank makes mistakes at a given policy decision, be it systematic or discretionary. Note that in addition to common shocks affecting all monetary union member countries, each individual country can experience idiosyncratic shocks which leads to different economic developments across the member countries. This implies that at any given point in time, the developments in different regions of the union might call for different policy responses. This highlights the importance of going beyond looking at the union level and looking at country level variables. In aggregating to the union level, the idiosyncratic developments across the member countries can wash out, potentially leading to policy prescriptions that may be optimal at the union level but suboptimal for a large part or even all of the member countries. This point gains further importance if there is heterogeneity in the causal effects of monetary policy on the

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[1] In these papers, optimal policy is defined as the policy that maximizes a utility based welfare criterion and is a function of the structural model parameters.

target variables across the member countries. Therefore, testing the optimality of monetary policy in a currency union has two dimensions, whether the policy actions are optimal from the perspective of the central bank, i.e., for the union as a whole, and whether they are optimal from the perspective of each individual country or blocks of countries. Additionally, performing such tests can be done retrospectively or contemporaneously. Ex-post testing of optimality can be of interest to researchers or policy makers in individual countries to assess whether past policy decisions have been optimal either from the standpoint of the monetary union or from the perspective of the individual country. Importantly, the test can also be implemented contemporaneously within a central bank during the policy decision making process to evaluate a given policy proposal.

In order to test whether monetary policy decisions are optimal a notion of optimality must be formalized. The starting point is the notion that the central bank should act to improve the welfare of the citizens of the union in which it acts. In the structural macroeconomic literature this is usually formalized by a utility based social welfare function which can be approximated by a quadratic form of all the variables in the model with a weighting matrix the entries of which are functions of the structural parameters of the model (Benigno and Woodford, 2012). However, in practice the central bank is charged with a mandate which typically involves only a few variables. This corresponds to substituting the weighting matrix in the quadratic form for a sparse weighting matrix that captures the central bank's mandate (Debortoli et al., 2019). This motivates defining optimal monetary policy based on a quadratic loss function of the variables specified in the central bank's mandate.

More specifically, this loss function is a discounted weighted sum of squared deviations of the paths of the per-country variables from their target. As the paths of the variables depend upon the actions of the central bank the gradient of the loss function with respect to the central bank's policy instruments should be zero if the policy decision is optimal. Furthermore, the gradient is a function of the expected path of the target variables in each country conditional on the current choice of the central bank's policy tools, and a measure of the causal effect of changes in the policy tools on the target variables in each country with the two weighed together based on the weight each country has in the loss function.

The central idea of the test statistic is then to combine these two ingredients to examine whether the central bank could lower its expected loss by deviating from its current policy plan. The exact form of the deviation chosen in the present paper has an interpretation as the first step in a Gauss-Newton optimization algorithm. Recall that in each iteration the Gauss-Newton algorithm informs about the direction, and gives a step size, in which the policy instruments should be changed to get closer to the optimal solution. In line with this, when the resulting perturbation does not equal zero the implication is that the current policy choice cannot be the optimal one. Furthermore, assuming that the policy instruments affect the target variables in a linear fashion, the resulting perturbation informs the policy maker by how much they must adjust each of their policy tools so that their policy choices conform with the optimal choice. Note however that this adjustment is a discretionary adjustment and that it does not inform the researcher how to change the systematic or rule-based part of monetary policy.

In practice, the conditional expectations of the policy maker are not readily available and one must rely on data that approximates the expectations. To that end, we use the policy maker's published forecasts for the target variables. Furthermore, the causal effects of the policy tools on the target variables must be estimated using statistical methods by the researcher as the policy maker does not publish their estimates of the causal effects. Given the potential estimation errors in the estimation of the causal effects and approximation errors when the conditional expectations are formed, we present a framework that takes this uncertainty into account and allows for making inference about the optimality of the current policy choice. This results in a confidence interval based on the quantiles of the approximating distribution of the evaluation statistic. This approximating distribution is based on simulations using the approximating distribution of the causal effect estimate and the approximating distribution of the conditional expectations of the policy maker. One can then reject the idea that the current policy choice is optimal if the confidence bands, calculated as the quantiles of the approximate distribution, do not include zero.

To illustrate the workings of the test statistic we first present intuition for the optimality condition implicit in the evaluation in a two country setting. The policy prescription of the optimality condition is that the optimal policy is the one which balances average future inflation between the two countries, corrected for the relative weight each country has in the central bank's mandate, the average ability of the policy instrument to transform a unit of inflation in one country into less inflation in another, and the dynamics of the transmission of monetary policy. A higher weight for a country implies that the central bank should maintain the target variables in that country closer to their target relative to other countries. Further, if the transmission mechanism of monetary policy is relatively worse in one country, the central bank should, again, keep deviations from target smaller in that country relative to the other. Finally, the relatively faster the transmission of monetary policy is in one country, the relatively smaller should the deviations from target be in that country at the optimum.

To further build intuition, we present a version of the simple stylized New Keynesian model of Benigno (2004) of a two country monetary union with a central bank that conducts monetary policy by setting the short term interest rate. To facilitate the comparison with the efficient solution of the model, we assume that the central bank's mandate coincides with the social welfare function implied by the model. We work through several versions of sub-optimal Taylor rules that could occur at a central bank in a monetary union, and show what the researcher would need to calculate if the model were known and the data truly generated by that model. We show how in all cases the test statistic can identify if monetary policy is not optimally conducted and how by adjusting the current choice of policy according to the prescription of the tool, the optimal outcome is achieved.

As an empirical application we provide a retrospective analysis of the policy decisions of the European Central Bank (ECB) whose primary objective is maintaining price stability in the Euro system.[2] In this case, the aggregate loss function is the discounted weighted sum

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[2] In theory, the framework could be used to test for optimality of U.S. monetary policy if one views the U.S. as a currency union of the 50 states. However, the Federal Reserve does not publish state level forecasts which makes the analysis impossible in practice.

of squared deviations of per country inflation from the ECB's inflation target of 2% where the weights correspond to the countries shares of overall euro area household final monetary consumption expenditure. To implement the test, the expected paths of the target variables and the causal effects are required. The paths of the target variables conditional on the policy choice are approximated by the forecasts published by the ECB around monetary policy meetings. These forecasts are available at the country level semi-annually from 2016Q2-2020Q2.

Additionally, we test the optimality of the ECB's monetary policy decisions at the member country level. Not rejecting optimality at the union level does not imply that the policy choice is optimal in every single member country. On the other hand, rejecting optimality at the union level does not mean that the policy choice is not optimal in any single member country. Testing optimality at the member country level can thus reveal potential heterogeneity among the member countries.

We further provide a case study of the monetary policy decision in 2020Q2, at the onset of the Covid-19 pandemic and subsequent economic crisis. Finally, we show how one can use the country level optimality test statistic, along with the union level test statistic, to provide a measure of the cost of giving up their independent monetary policies for each member country.

The ECB conducts its monetary policy through interest rate policies intended to affect the shape of the yield curve which in turn affects the economic decisions of firms and households in the union and thus the rates of inflation in the member countries. In the present paper, we consider three policy instruments that the ECB has at its disposal. Target Rate policies which affect the short end of the yield curve, Forward Guidance policies which affect the slope of the yield curve over the medium horizon, and the recently introduced Quantitative Easing policies intended to affect the long end of the yield curve. In order to obtain the causal effects of each of these instruments on the per-country inflation rates we proceed in two main steps. Using the database provided by Altavilla et al. (2019) and following the methodology therein we construct proxies for monetary policy surprises for the three policy tools based on changes in prices of various euro area assets at various maturities in a window around the policy announcements. We then estimate the country level causal effects of changes in the policy tools using instrumental variable local projections for each member country and each policy tool with the proxies estimated in the first step acting as the instruments.

The results from the test indicate that there are multiple suboptimalities in the ECB's policy decisions. While we can never reject optimality of the ECB's decisions regarding the Target Rate at the union level, we can reject optimality in the ECB's use of Forward Guidance in 6 out of 9 decisions and for Quantitative Easing we can reject optimality in all 9 decisions. In all cases of rejection, for both the Forward Guidance and the Quantitative Easing instruments, the test statistic indicates that the ECB should have used its policy to lower both the slope of the yield curve and the long end of the yield curve and thus provide more stimulus to support the Euro area economy.

When looking at the contribution of the individual member countries to the union level statistic we find a fair bit of heterogeneity when it comes to the Target Rate instrument and, to a lesser extent, for the Forward Guidance instrument. Furthermore, the sign of the union level statistic for the Target Rate instrument always matches the sign of the test statistic for Germany. This is not the case for the other largest member countries with the test statistics for France and Italy often having the opposite sign to the union level statistic. In about half of the cases the sign of the contribution of the smallest 14 member countries has the opposite sign to the union level statistic. For the Forward Guidance instrument there is some heterogeneity in two of the decisions with the largest economies being split in the prescriptions that would make policy optimal for their economies. Interestingly, as opposed to the result for the Target Rate, the sign of the test statistic for Germany is different from the union level statistic in four cases. With regard to the Quantitative Easing instrument, there is only one instance of one of the five largest economies having a test statistic with a different sign than the union level statistic, revealing substantial homogeneity when it comes to the prescriptions for that instrument.

Despite the fact that we can never reject optimality for the Target Rate instrument at the union level, we can, in fact, reject optimality in the decisions for the Target Rate for several member economies in some instances. The number of countries in which we can reject optimality ranges from 1 to 3 per decision with the exception of 2020Q2 when the number rises to 5. The highest frequency of rejection is in Finland, in which we can reject optimality in 6 out of the 9 decisions.

As hinted at before, in the case of the Forward Guidance instrument, there is heterogeneity in the direction of the adjustments necessary to restore optimality in the instances where we can reject. The number of countries in which we can reject optimality ranges between 4 and 16 per decision. The worst results are for Ireland in which we can reject optimality in every period. Additionally, when we can reject optimality at the union level, there are countries in which we can also reject optimality but where the sign of the optimal adjustment is opposite to the sign of the optimal union level adjustment. However, in every instance we can reject optimality at the union level, Italy is in the set of countries for which we can reject optimality at the country level and in all cases does the sign of the optimal adjustment for Italy match the sign of the optimal adjustment at the union level.

Despite the fact that we can reject the optimality of the setting of the Quantitative Easing instrument in every period, the number of countries in which we can reject optimality ranges between only 2 to 8 per decision. The worst results are in the case of Portugal in which we can reject optimality at all decisions. The set of 9 countries for which we can reject optimality of the Quantitative Easing instrument includes 8 out of the 10 countries with highest public debt to GDP ratio in the Euro area with the 9th country having the 16th highest public debt to GDP ratio. The set also includes 5 countries which received a bailout package during the European Sovereign Debt Crisis.

The final empirical exercise performed is to provide a measure of the cost in each member country of giving up their independent monetary policy. This is calculated as the ratio of the per country expected loss, after adjusting policy by the per country optimal adjustment, to the expected loss in each country after adjusting policy by the union level optimal adjustment. We find that the median reduction in expected loss from having independent monetary policy is 58% with the smallest reduction in Spain at 4% but the largest reduction in Finland at 92% lower expected loss. The asynchronous economies of Estonia, Lithuania, Latvia and Slovakia could lower their expected loss by 85-91% with independent monetary policies as the business cycles of these countries are weakly synchronized with the Euro area business cycle and, in fact, their loss would increase if the union level policy is adjusted by the optimal adjustment relative to no adjustment at all. Finally, 12 out of the 19 member economies could reduce their expected loss by more than 50% by conducting monetary policy independently, with the set of these countries indicating that the highest price of Euro membership is paid by the peripheral economies.

The remainder of the paper is organized as follows. After reviewing the related literature in the next subsection, Section 2 presents the testing environment, the test statistic, how to perform inference, and the country level perspective of the testing framework. Section 3 provides the intuition implicit in the test statistic using a two country framework and a structural model. Section 4 illustrates how to utilize the framework for the ECB and Section 5 presents the results of a retrospective analysis of the optimality of the ECB's decisions at the union and country levels along with the measure of the cost of joining the monetary union. Section 6 concludes.

1.1 Related Literature

With the exception of Barnichon and Mesters (2023), who present a framework for testing the optimality of decisions made by a central bank responsible for monetary policy within a single country, the literature on testing for optimality of monetary policy is non-existent. They label their test statistic the Optimal Policy Perturbation or OPP. The present paper builds on their work to answer the previously unaddressed question of how to test for optimization failures in a currency union.

Additionally, the present paper lies at the intersection of several strands of literature. Firstly, the paper is related to the extensive literature on optimal policy making (see, e.g., the textbooks of Ljungqvist and Sargent, 2004, Woodford, 2003, Galí, 2015, or Bénassy-Quéré et al., 2018). In particular, the literature on optimal policy making in a monetary union (see, e.g., Benigno, 2004, Galí and Monacelli, 2008, Bhattarai, Lee and Park, 2015). However, the paper deviates from that literature in that rather than searching for the functional form of an optimal policy rule, the OPP is designed to test for optimization failures. In that sense, the present paper should be seen as complementary to the literature on optimal policy making.

With the starting point of the OPP being a high-level macroeconomic welfare function specified by the policy maker rather than a utility based social welfare function relates the paper back to the work of Tinbergen (1952). Taking a step away from micro-founded structural models of the macroeconomy is beneficial in several ways. As discussed, e.g., in Svensson (2003) and Blanchard (2016), model misspecification is a real concern when it comes to monetary policy making and in fact, policy makers must often make decisions based on heuristic arguments, weigh together the different prescriptions of different models, and make judgment calls. Furthermore, in many cases, the underlying economic model is simply too complicated to be written down without incurring prohibitively high costs. The fact that the OPP does not build on a specific micro-founded model is its main strength as it is removed from the debate on the exact functional form of the underlying economic model. In that sense the approach in the present paper is in the spirit of Hansen and Sargent (2008) in attempting to be robust to model misspecification. Furthermore, as mentioned in Barnichon and Mesters (2023), the OPP manages to preserve the simplicity and transparency of the Tinbergen rule while generalizing it to a dynamic context.

The OPP is closely related to the sufficient statistics approach to welfare analysis in public finance (see, e.g., Chetty, 2009, Kleven, 2020). Both ideas build on the fact that the welfare effects of a policy can be calculated based on high-level elasticities, in the present paper the causal effects of changes in the ECB's policy instruments on the rate of inflation in the euro area member countries. The present approach diverges from the literature on public finance in that the OPP does not require any envelope condition and therefore knowledge of the true underlying economic model is not required.

While the OPP is not intended to dictate how the monetary authorities should conduct their actions but to test for optimization failures, the OPP is linked to the literature on inflation forecasting rules for monetary policy (e.g., Svensson, 1997, 1999, 2017, 2020, Woodford, 2007). Inflation forecasting rules state that a central bank should set an entire path for their policy instruments such that forecasts of the target variables are in compliance with the central bank's mandate, i.e., that the target variables reach their target within an appropriate time frame. The OPP provides a quantifiable framework for this philosophy of thought and can aid a central bank in the communication of such policy decisions.

Finally, the paper is related to the general literature on structural impulse response in macro-econometrics (see, e.g., Ramey (2016)). However, as opposed to the standard practice of using the estimated responses to guide model building, as in Christiano, Eichenbaum and Evans (2005), the paper continues the work of Barnichon and Mesters (2023) of using the impulse response function to build a test statistic for optimality of monetary policy. The paper uses the tools presented in the literature on local projections estimated with external instruments (see, Jordà, 2005, Stock and Watson, 2018) where the external instruments are estimated from high-frequency price changes in assets around monetary policy decisions (see, e.g., Altavilla et al., 2019, Kuttner, 2001, Gürkaynak, 2005, Swanson, 2017). In that sense, the paper is related to the literature on the estimation of the causal effects of monetary policy in the euro area (see, e.g., Corsetti, Duarte and Mann, 2020, Jarocinski and Karadi, 2020, Andrada and Ferroni, 2020, Schrimpf, 2019, or Brand, Buncic and Turunen, 2010) and the potentially heterogeneous effects of ECB monetary policy across the euro area member countries (see, e.g., Corsetti, Duarte and Mann, 2020, Burriel and Galesi, 2018, Gavilan-Rubio, 2019, Mandler, Scharnagl and Volz, 2016, Ciccarelli, Maddaloni and Peydró, 2013).

2 Testing Optimality in a Monetary Union

2.1 Environment

The central bank of a monetary union is, as with most central banks, given a set of mandates from society which are deemed relevant for improving, if not maximizing, the welfare of said society. These mandates involve in almost all cases some measure of price stability, e.g., an inflation target, and sometimes these mandates are expanded to include more general macroeconomic stabilization goals such as maximizing employment without jeopardizing price stability. Furthermore, these mandates are often defined over a specific time horizon to allow temporary deviations if they are indeed temporary and to allow for gradual reaction to any deviations if reacting fully and aggressively to stabilize one objective could cause excessive costs in other areas of the economy.

To select the loss function for the union's central bank we start by noting that based on the work by Benigno and Woodford (2012), we have that for a given macroeconomic model, a utility based social welfare function can be approximated by a quadratic form in all of the model's variables, with an appropriate weighting matrix based on the model's structural parameters. As noted in Debortoli et al. (2019) a central bank is then given a mandate involving only a few of the model's variables and that this mandate corresponds to substituting the weighting matrix in the welfare function for a sparse matrix with entries that correspond to the mandate. This motivates representing the mandates of a central bank in a currency union as a quadratic loss function in the variables specified in its mandate.

At any given point in time the central bank aims to conduct monetary policy such that it minimizes the expected deviations of the discounted paths of its target variables from the policy targets. For a central bank in a monetary union with N member countries and M mandates, denote these paths by $Y_t = (Y'_{1,t}, \ldots, Y'_{N,t})'$ where $Y_{i,t} = (y'_{i,t}, y'_{i,t+1}, \ldots)'$ where $y_{i,t} = (y_{i,t,1}, \ldots, y_{i,t,M})'$. The expectation regarding the future path of Y_t based on the information set $\mathcal{F}_t$ at time t is captured by $\mathbb{E}_t Y_t = \mathbb{E}(Y_t | \mathcal{F}_t)$.

The central bank thus aims to use its policy tools to minimize the expected loss function

$$\mathcal{L}_t = \mathbb{E}_t Y'_t \Omega Y_t \tag{1}$$

where $\Omega = \operatorname{diag}(\beta \otimes (\omega \otimes \nu))$ where $\omega = [\omega_i]_{i=1,\ldots,N}$, $\nu = [\nu_m]_{m=1,\ldots,M}$, and $\beta = [\beta_h]_{h=0,\ldots,H}$ where β_h is the discount factor at horizon h, ν_m is the weight attached to mandate m, ω_i is the weight that country i receives in the mandate. This loss function emphasizes that the framework adapts easily to many mandates and countries and that the mandates could be country specific.

As in Barnichon and Mesters (2023) we will assume that the underlying economy can be characterized by a generic linear model where the non-policy block of the economy at time t is given by

$$\begin{aligned} \mathcal{A}_{yy}\mathbb{E}_t Y_t - \mathcal{A}_{yz}\mathbb{E}_t Z_t - \mathcal{A}_{yp}P_t^e &= \mathcal{B}_{y\Lambda}\Lambda_{-t} + \mathcal{B}_{y\xi}\mathbb{E}_t\Xi_t \\ \mathcal{A}_{zz}\mathbb{E}_t Z_t - \mathcal{A}_{zy}\mathbb{E}_t Y_t - \mathcal{A}_{zp}P_t^e &= \mathcal{B}_{z\Lambda}\Lambda_{-t} + \mathcal{B}_{z\xi}\mathbb{E}_t\Xi_t \end{aligned} \tag{2}$$

where $Z_t = (z_t', z_{t+1}', \ldots)'$ is the path of the endogenous variables less the target variables in Y_t that are relevant for the determination of the paths of the target variables in the member countries of the monetary union. Note that the model allows for different variables affecting the target variables in each member country and allows for spillover effects from the endogenous variables between countries. $\Lambda_{-t} = (y_{t-1}', z_{t-1}', p_{t-1}', y_{t-2}', \ldots)'$ contains the initial conditions defined by the past path of the variables y_t, z_t and p_t, and $\Xi_t = (\xi_t', \xi_{t+1}', \ldots)'$ is the path of structural shocks. These structural shocks can potentially have a factor structure and include common union shocks or shocks specific to blocks of member countries in addition to country specific idiosyncratic shocks.

The policy maker's choice

To achieve its mandate, the central bank is given a set of policy instruments which it uses to steer the target variables towards their targets. In the context of monetary policy these instruments correspond to tools intended to affect the shape of the yield curve, the central bank can set the short-term rate, conduct forward guidance, buy and sell long-maturity securities, and engage in Quantitative Easing programs. Here we will evaluate the central bank's use of three policy tools, the contemporaneous policy rate (the Target Rate), the slope of the expected path of the policy rate (Forward Guidance), and its use of Quantitative Easing to minimize the loss function.[3] Denote the expected path of policy by $P^e_t = \mathbb{E}_t(p'_t, p'_{t+1}, \ldots)'$ where $p_t = (TR_t, FG_t, QE_t)'$ where TR_t is the Target Rate, FG_t is Forward Guidance, which affects the medium run slope of the yield curve, and QE_t is Quantiative Easing, which affects the long end of the yield curve. One can think of p_t as a parsimonious characterization of the path of the yield curve and thus that P^e_t captures expectations on how the yield curve will evolve over time. For example, $\mathbb{E}_t(FG_{t+h})$ is the expectation of the medium run slope of the yield curve at time $t+h$ given information at time t.

We define the optimal policy $P^{e,opt}_t$ as the policy path chosen by a planner solving the problem

$$\min_{Y_t, Z_t, P_t} \mathcal{L}_t \quad \text{s.t. } (2) \tag{3}$$

The practical policy decision is assumed to be composed of two parts, the policy rule which captures the response to all available time-t measurable variables and an exogenous component. We assume a generic model for the policy block of the form

$$\mathcal{A}_{pp} P^e_t - \mathcal{A}_{py} \mathbb{E}_t Y_t - \mathcal{A}_{pz} \mathbb{E}_t Z_t = \mathcal{B}_{p\Lambda} \Lambda_{-t} + \mathcal{B}_{p\xi} \mathbb{E}_t \Xi_t + \varepsilon^e_t \tag{4}$$

where $\varepsilon^e_t = \mathbb{E}_t \varepsilon_t$ are shocks to the expected policy paths with $\varepsilon_t = (\epsilon'_t, \epsilon'_{t+1}, \ldots)'$ where $\epsilon_t = (\epsilon'_{TR,t}, \epsilon'_{FG,t}, \epsilon'_{QE,t})'$. Note that taking the expectation of ε_t transforms the shocks into policy news shocks at time t which are assumed to be uncorrelated with the initial conditions and all other structural shocks.

We summarize the parameters of the policy rule as $\phi = \{\mathcal{A}_{pp}, \mathcal{A}_{py}, \mathcal{A}_{pz}, \mathcal{B}_{p\Lambda}, \mathcal{B}_{p\xi}\}$ and define a policy choice as P^e_t which is determined by the pair (ϕ, ε^e_t). The Fed's proposed expected policy path is denoted by P^{e0}_t, determined by the pair $(\phi^0, \varepsilon^{e0}_t)$. Our interest lies in testing if $P^{e0}_t = P^{e,opt}_t$.

If we assume that the optimal policy $P^{e,opt}_t$ is unique and that the rule ϕ^0 leads to a

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[3] Limiting the evaluation to these three policy tools and thus ignoring other possible tools is why the present discussion is on the sub-set OPP statistic of Barnichon and Mesters (2023) who also present a more general framework with M_p number of possible policy tools.

unique and determinate equilibrium Barnichon and Mesters (2023) show that

$$P_t^{e0} = P_t^{e,opt} \quad \Longleftrightarrow \quad \nabla_{\varepsilon_t} \mathcal{L}_t|_{P_t^{e0}} = \mathcal{R}^{0\prime} \Omega \mathbb{E}_t Y_t^0 = 0 \tag{5}$$

where $\mathbb{E}_t Y_t^0$ is the allocation given P_t^{e0} and $\mathcal{R}^0$ measures the causal effects of the policy shocks to the target variables under the rule ϕ^0. Note that under the rule ϕ^0 the proposed policy will lead to a unique equilibrium and therefore we can write the model of (2) and (4) as

$$\begin{aligned} \begin{bmatrix} \mathbb{E}_t Y_t \\ \mathbb{E}_t Z_t \\ P_t^e \end{bmatrix} &= \begin{bmatrix} \mathcal{A}_{yy} & -\mathcal{A}_{yz} & -\mathcal{A}_{yp} \\ -\mathcal{A}_{zy} & \mathcal{A}_{zz} & -\mathcal{A}_{zp} \\ -\mathcal{A}_{py} & -\mathcal{A}_{pz} & \mathcal{A}_{pp} \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{B}_{y\Lambda} & \mathcal{B}_{y\xi} & 0 \\ \mathcal{B}_{z\Lambda} & \mathcal{B}_{z\xi} & 0 \\ \mathcal{B}_{p\Lambda} & \mathcal{B}_{p\xi} & I \end{bmatrix} \begin{bmatrix} \Lambda_{-t} \\ \mathbb{E}_t \Xi_t \\ \varepsilon_t^e \end{bmatrix} \\ &= \begin{bmatrix} \mathcal{C}_{y\Lambda}^0 & \mathcal{C}_{y\xi}^0 & \mathcal{R}^0 \\ \mathcal{C}_{z\Lambda}^0 & \mathcal{C}_{z\xi}^0 & \mathcal{C}_{z\varepsilon}^0 \\ \mathcal{C}_{p\Lambda}^0 & \mathcal{C}_{p\xi}^0 & \mathcal{C}_{z\varepsilon}^0 \end{bmatrix} \begin{bmatrix} \Lambda_{-t} \\ \mathbb{E}_t \Xi_t \\ \varepsilon_t^e \end{bmatrix} \end{aligned} \tag{6}$$

Note that using the first equation of the system, under P_t^{e0} we can write

$$Y_t^0 = \mathcal{R}^0 \varepsilon_t^{e0} + \underbrace{\mathcal{C}_{y\Lambda}^0 \Lambda_{-t} + \mathcal{C}_{y\xi}^0 \mathbb{E}_t \Xi_t + Y_t^0 - \mathbb{E}_t Y_t^0}_{\Upsilon_t^0} \tag{7}$$

where Υ_t^0 is a linear combination of the model's structural shocks, its initial conditions and any future errors $Y_t^0 - \mathbb{E}_t Y_t^0$ and $\mathbb{E}(\varepsilon_t^{e0} \Upsilon_t^0) = 0$ follows from the assumption that the time t policy news shocks ε_t^{e0} are orthogonal to the initial conditions and the other structural shocks. Therefore, if one has a measure of the policy news shocks, one could estimate the causal effects $\mathcal{R}^0$ using local projections as we will do in a later section.

2.2 Test Statistic

Going back to the gradient in (5), if we rescale it by the inverse Hessian matrix we arrive at a monetary union version of the OPP statistic of Barnichon and Mesters (2023)

$$\delta_t^\star = -(\mathcal{R}^{0\prime}\Omega\mathcal{R}^0)^{-1}\mathcal{R}^{0\prime}\Omega\mathbb{E}_t Y_t^0 \tag{8}$$

which, under the current set up, has the property that $\delta_t^\star = 0$ if and only if $P_t^{e0} = P_t^{e,opt}$ and, furthermore, $P_t^{e0} + \delta_t^\star = P_t^{e,opt}$. Testing whether policy is optimally set for the monetary union thus boils down to testing whether $\delta_t^\star$ is statistically different from zero.

Note that the OPP statistic relies primarily on two objects, the expected path of the target variables $\mathbb{E}_t Y_t^0$, and the matrix of causal effects $\mathcal{R}^0$. As we assume that the true structure of the economy (2) and (4) is not known we need a way to estimate or approximate the causal effects and the expected paths. As returned to later, given Equation (7) we can use local projections to estimate the elements of the matrix of causal effects and since the expected path of the target variables corresponds to the central bank's expectations for the target variables conditional on the bank's choice for the policy instruments we can use forecasts published by the central bank to approximate the expected path $\mathbb{E}_t Y_t^0$. Importantly, to test whether policy is optimally set we do not need to assume that the researcher has access to the true underlying model of the economy for the member countries.

2.3 Inference

When it comes to the practical implementation of the test there are two sources of uncertainty. The first is that the true causal effects $\mathcal{R}$ are not known to the researcher and must thus be estimated. The second is that the policy maker is unable to calculate the optimal forecasts $\mathbb{E}_t Y_t^0$ and therefore its forecasts are only approximations of the conditional expectations. Taking these sources of uncertainty into account, we present a way of calculating confidence bounds for the test statistic.

Leaving the details of the estimation for a later section, for now we assume that $\mathcal{R}$ can be estimated. Let $r = \text{vec}(\mathcal{R})$ and denote its estimate by $\hat{r}$. For now, we will assume that we can approximate the distribution of r by

$$\hat{r} \sim N(r, \Sigma_r)$$

where Σ_r is the variance matrix of all impulse responses across all horizons, mandates and countries. This normality assumption can be justified in the frequentist sense by asymptotic arguments since when the sample size tends to infinity, several popular estimators are asymptotically normally distributed. An alternative situation could arise within a central bank where the researcher performing the test of optimality does not need to estimate these causal effects but rather is provided with estimates of the parameters of the distribution for r from within the central bank. Importantly, the calculation of the test statistic does not depend on the specific method used to estimate the causal effects, nor in fact on the approximating distribution being a Gaussian distribution. Regardless of how the approximating distribution is obtained, denote by $r_t^{(j)}$ a draw from this approximate distribution.

Turning to the uncertainty surrounding the conditional expectation of the target variables, $\mathbb{E}_t Y_t^0$, let us denote the central bank's stacked vector of per country forecasts by $\hat{Y}_{t|t}$ which are its approximation of the conditional expectation. This forecast is an approximation to the conditional expectations and is usually based on a suite of models and expert judgment. As such, the forecasts are subject to misspecification issues that need to be taken into account. Again, deferring the specifics to a later section, assume that we can approximate the distribution of the conditional expectation $\mathbb{E}_t Y_t^0$ as

$$\mathbb{E}_t Y_t^0 \sim N(\mu_{Y,t}, \Sigma_{Y,t|t}) \tag{9}$$

where $\mu_{Y,t}$ is the mean of the distribution and $\Sigma_{Y,t|t}$ denotes the covariance matrix. Denote by $Y_{t|t}^{(j)}$ a draw from this approximate distribution.

With the approximating distributions of the misspecification error and causal effects we can construct confidence intervals for the test statistic using simulation to approximate the distribution of $\delta_t^\star = -(\mathcal{R}_t'\Omega\mathcal{R}_t)^{-1}\mathcal{R}_t'\Omega\mathbb{E}_tY_t^0$. we compute

$$\delta_t^{(j)} = -(\mathcal{R}_t^{(j)\prime}\Omega\mathcal{R}_t^{(j)})^{-1}\mathcal{R}_t^{(j)\prime}\Omega Y_{t|t}^{(j)}, \quad j = 1, \ldots, B \tag{10}$$

where $r_t^{(j)} = \text{vec}(\mathcal{R}^{(j)})$ is a draw from the approximate distribution of the impulse response functions and $Y_{t|t}^{(j)} \sim N(\hat{Y}_{t|t}, \hat{\Sigma}_{Y,t|t})$ and B is the number of draws from the approximating distributions. The empirical section below reports median and upper and lower bounds of the simulated distribution $\{\delta_t^{(j)}, j = 1, \ldots, B\}$ at each point in time. The decision rule we will follow is that we will reject the hypothesis that the policy decision is optimal if zero lies outside the 68% confidence bounds calculated as the 16th and 84th quantiles of the simulated distribution of $\delta_t^\star$.

2.4 Country level perspective

The multicountry aspect of the current framework provides the opportunity to further define optimization failures from the perspective of each individual country. Begin by assuming that there exists a policy choice $P_{i,t}^{e,opt}$ that minimizes the country specific loss function, i.e. that solves the problem

$$\min_{Y_{i,t}, Z_{i,t}, P_{i,t}} \mathcal{L}_{i,t} = \mathbb{E}_t Y_{i,t}'\Omega_i Y_{i,t} \quad \text{s.t. } (2) \tag{11}$$

where $\Omega_i = \text{diag}(\beta \otimes \nu)$ is the country level specific weighting matrix. If we again assume that the policy choice $P_{i,t}^{e,opt}$ is unique and that the rule ϕ^0 leads to a unique and determinate equilibrium, we have that

$$P_{i,t}^{e0} = P_{i,t}^{e,opt} \quad \Longleftrightarrow \quad \nabla_{\varepsilon_t}\mathcal{L}_{i,t}|_{P_{i,t}^{e0}} = \mathcal{R}_i^{0\prime}\Omega_i\mathbb{E}_tY_{i,t}^0 = 0 \tag{12}$$

which is the country level equivalent to Equation (5). This further leads to a member country version of the OPP statistic

$$\delta^{\star}_{i,t} = -(\mathcal{R}^{0\prime}_i \Omega_i \mathcal{R}^0_i)^{-1} \mathcal{R}^{0\prime}_i \Omega_i \mathbb{E}_t Y^0_{i,t} \tag{13}$$

which has comparable properties to the monetary union version above, i.e., $\delta^{\star}_{i,t} = 0$ if and only if $P^{e0}_{i,t} = P^{e,opt}_{i,t}$ and, furthermore, $P^{e0}_{i,t} + \delta^{\star}_{i,t} = P^{e,opt}_{i,t}$. Testing whether policy is optimally set for a member country thus boils down to testing whether $\delta^{\star}_{i,t}$ is statistically different from zero.

An important question economically as well as politically is how does this policy optimality condition for the currency union's central bank relate to policy optimality from the perspective of each member country, i.e., what would be the optimal policy response of a national central bank only charged with conducting monetary policy within their respective countries. As the central bank of a currency union needs to weigh together the different policy needs of the individual countries, combined with the fact that the individual countries are subjected to idiosyncratic shocks which can lead to divergent economic developments, the central bank's actions will in general not be optimal from the standpoint of each individual country.

To formalize the idea that a central bank in a currency union must balance the deviations of the target variables in each of the member countries note that since Ω is diagonal, we can write

$$\begin{aligned}
\delta^{\star}_t &= -(\mathcal{R}'_t \Omega \mathcal{R}_t)^{-1} \mathcal{R}'_t \Omega \mathbb{E}_t Y^0_t \\
&= -(\mathcal{R}'_t \Omega \mathcal{R}_t)^{-1} (\mathcal{R}'_{1,t} \Omega_1 \mathbb{E}_t Y^0_{1,t} + \ldots + \mathcal{R}'_{N,t} \Omega_N \mathbb{E}_t Y^0_{N,t}) \\
&= W \mathcal{R}'_{1,t} \Omega_1 \mathcal{R}_{1,t} \delta^{\star}_{1,t} + \ldots W \mathcal{R}'_{N,t} \Omega_N \mathcal{R}_{N,t} \delta^{\star}_{N,t} \\
&= W_1 \delta^{\star}_{1,t} + \ldots + W_N \delta^{\star}_{N,t}
\end{aligned}$$

where $W = (\mathcal{R}'_{1,t} \Omega_1 \mathcal{R}_{1,t} + \ldots + \mathcal{R}'_{N,t} \Omega_N \mathcal{R}_{N,t})^{-1}$, $W_i = W \mathcal{R}'_{i,t} \Omega_i \mathcal{R}_{i,t}$ is a country specific weighting matrix, and $\delta^{\star}_{i,t} = (\mathcal{R}'_{i,t}\Omega_i\mathcal{R}_{i,t})^{-1}\mathcal{R}'_{i,t}\Omega_i\mathbb{E}_tY^0_{i,t}$ is the optimal adjustment of the policy instruments if the central bank would only care about country i, i.e., the optimal adjustment of a national central bank whose mandate involves the same target variables as the currency unions central bank, but only at the national level. This makes clear that the optimal adjustment for a central bank in a currency union whose mandate involves multiple countries, is a weighted average of the optimal adjustments of each individual member country. Further note that within $\delta^{\star}_{i,t}$, $\Omega_i = \mathrm{diag}(\beta \otimes \nu)$, that is, the country weight does not enter $\delta^{\star}_{i,t}$ but is present in W_i.

Note that the current setup never assumes that $P^{e,opt}_{i,t} = P^{e,opt}_{t}$. This highlights the fact that joining a monetary union and giving up having an independent domestic monetary policy exposes the economy to a possible externality caused by the economic situation in other member countries calling for a different policy response than the country would choose for itself. Thus, it is possible that joining a monetary union precludes the member country from the possibility of monetary policy being conducted optimally from its own standpoint.

This emphasizes the role other economic policies must play in supporting the union level monetary policy both at the union level but even more so at the member country level. The larger is the deviation of the target variables from their target at the country level, the larger would be the required monetary policy response. However, as long as the other member countries are not experiencing similar deviations of their target variables, the currency union central bank's policy response will be smaller than what would be optimal at the country level. Therefore, other economic policies, such as tax or government spending policies, must step in to fill the gap and support the monetary policy decision with policies that support aligning the target variables with their respective targets.

Note that in the preceding discussion the underlying assumption is that the national central bank has the same preferences across the different mandates and the same time discount factor as the currency union central bank. It is of course possible that if the individual country were not part of the currency union that it would give its central bank a different mandate in terms of the weights each target variable receives, the numerical values of the targets, and what the target variables are. Given that the currency union central bank does not react fully to any individual member state, there remains the possibility that deviations of the target variables from the targets in individual countries is in fact due to country level policies that aim to bring that country's economic outcomes closer to the targets a country level central bank with a different mandate from the currency union central bank, reflecting the different preferences of that country. This concern is, however, outside the scope of the present paper. Furthermore, the empirical application below uses the euro area in which each member country has opted into joining and therefore it can be argued that the mandates a national central bank would be given is close to the mandate of the European Central Bank. Nevertheless, the concept of a national central bank in the preceding discussion should be thought of as a central bank that has the same mandate as the currency union central bank except only defined for that specific country.

3 Intuition

In order to clarify the intuition and functioning of the test statistic the present section provides examples in two different two-country settings. The first assumes that the central bank has a single mandate of stabilizing inflation in the two countries and shows what the prescription of the test statistic implies for the target variables in the two countries and builds some simple economic intuition. The second presents calculations based on a structural model showing what the OPP would calculate if the model, and thus the data generating process and the societal loss function, were known.

3.1 What the optimality condition implies

Consider the case when the monetary union consists of two countries, A and B, the central bank has the unique mandate of stabilizing inflation, and assume for simplicity that $\beta_h = 1$, for $h = 0, \ldots, H$. In this case we can write the central bank's loss function as

$$\mathcal{L}_t = \mathbb{E}_t \sum_{h=0}^{H} \left[\omega(\pi_{A,t+h} - \pi^\star)^2 + (1-\omega)(\pi_{B,t+h} - \pi^\star)^2\right] \tag{14}$$

$$= \omega \mathbb{E}_t \sum_{h=0}^{H} (\pi_{A,t+h} - \pi^\star)^2 + (1-\omega)\mathbb{E}_t \sum_{h=0}^{H} (\pi_{B,t+h} - \pi^\star)^2 \tag{15}$$

$$= \omega E_t ||\Pi_t^{(A)}||^2 + (1-\omega) E_t ||\Pi_t^{(B)}||^2 \tag{16}$$

where $\Pi_t^{(i)} = (\pi_{i,t} - \pi^\star, \pi_{i,t+1} - \pi^\star, \ldots, \pi_{i,t+H} - \pi^\star)$ and ω is the weight assigned to country A in the central bank's mandate.

The necessary condition in Equation 5 can be rewritten to illustrate the workings of the test statistic. In particular, the necessary condition for monetary policy optimality with respect to a policy instrument $p_{k,t}$, i.e., $\delta^\star_{k,t} = 0$ is equivalent to

$$||\mathbb{E}_t \Pi_t^{(A)}|| = \frac{(1-\omega)}{\omega} \frac{1}{\kappa_{k,t}} \frac{1}{\chi_{k,t}} ||\mathbb{E}_t \Pi_t^{(B)}|| \tag{17}$$

where

$$\kappa_{k,t} = \frac{||\mathcal{R}_k^{(B)}||}{||\mathcal{R}_k^{(A)}||} \quad and \quad \chi_{k,t} = -\frac{\rho_{k,t}^{(A)}}{\rho_{k,t}^{(B)}} \quad for \quad k = 1 \ldots K \tag{18}$$

where $\rho_{k,t}^{(i)} = \frac{\mathbb{E}_t \mathcal{R}_k^{(i)\prime} \Pi_t^{(i)}}{||\mathbb{E}_t \Pi_t^{(i)}|| \; ||\mathcal{R}_k^{(i)}||}$ denotes the uncentred correlation between $\mathbb{E}_t \Pi_t^{(i)}$ and $\mathcal{R}_k^{(i)}$, where $\mathcal{R}_k^{(i)}$ is the k-th column of $\mathcal{R}^{(i)}$ for $i = A, B$.

This illustrates how optimally conducting policy involves balancing expected future deviations of inflation from target between the two countries where the paths of future deviations are summarized according to the ℓ_2-norm. The parameters $\kappa_{k,t}$, ω, and $\chi_{k,t}$ determine the exact nature of the tradeoff between balancing inflation in each country. Firstly, ω determines the importance the union attaches to stabilizing inflation in each country. For example, this could be determined by the relative population size or the countries share in total consumption. Secondly, $\kappa_{k,t}$ captures the average ability of instrument k to transform one unit of inflation in country B into less inflation in country A. Thirdly, $\chi_{k,t}$ adjusts $\kappa_{k,t}$ to take into account the dynamics of the transmission of monetary policy as monetary policy instruments generally affect the economy with a lag, often not reaching their full effect until several quarters after the instrument being used.

An alternative presentation yields a more economic interpretation. The optimality condition in Equation 17 can be rewritten as equalizing the monetary union's marginal rate of transformation between average inflation in country A and country B and the marginal rate of transformation between average inflation in country A and country B for policy instrument k. That is, for if policy instrument $p_{k,t}$ is optimally set, we must have that

$$MRS_t = MRT_{k,t} \tag{19}$$

where

$$\begin{aligned} MRS_t &= \frac{\partial L_t}{\partial ||\mathbb{E}_t \Pi_t^{(A)}||} / \frac{\partial L_t}{\partial ||\mathbb{E}_t \Pi_t^{(B)}||} = \frac{\omega ||\mathbb{E}_t \Pi_t^{(A)}||}{(1-\omega)||\mathbb{E}_t \Pi_t^{(B)}||} \\ MRT_{k,t} &= \frac{\partial ||\mathbb{E}_t \Pi_t^{(B)}||}{\partial \delta_{k,t}} / \frac{\partial ||\mathbb{E}_t \Pi_t^{(A)}||}{\partial \delta_{k,t}} = \frac{1}{\kappa_{k,t} \chi_{k,t}} \end{aligned}$$

The implication is that a central bank that is conducting its policy optimally should balance the welfare cost of allowing relative average inflation in both countries deviate from the inflation target, the MRS, with the cost of monetary policy actions, the MRT. Rewriting Equation (19) as

$$\frac{||\mathbb{E}_t \Pi_t^{(A)}||}{||\mathbb{E}_t \Pi_t^{(B)}||} = \frac{(1-\omega)}{\omega} \frac{1}{\kappa_{k,t} \chi_{k,t}} \tag{20}$$

yields several insights about the optimal balance of deviations from the inflation target between the two countries. First, as expected, we see that the higher the weight ω, the more effort is put into minimizing deviations in country A relative to country B. Second, the less responsive inflation in country A is to changes in the policy instrument, as measured by $\|\mathcal{R}_{k,t}^{(A)}\|$, the higher the value of $\kappa_{k,t}$ and thus the lower is the optimal ratio $\frac{||\mathbb{E}_t \Pi_t^{(A)}||}{||\mathbb{E}_t \Pi_t^{(B)}||}$. That is, the harder it is for the central bank to steer the inflation rate towards target in a specific country, the more emphasis it should place on minimizing the deviation of inflation from target in that country. This rhymes well with the standard result in structural macroeconomics that a central bank should focus more on stabilizing inflation in sectors or countries where inflation is more persistent as a higher degree of price stickiness leads to smaller impulse responses of inflation to monetary policy shocks. Finally, the faster is the transmission of monetary policy in country A, i.e., the higher is the uncentred correlation between the expected path of inflation and the impulse responses $\rho_{k,t}^{(A)}$, the larger is $\chi_{k,t}$ in absolute value implying a lower optimal ratio $\frac{||\mathbb{E}_t\Pi_t^{(A)}||}{||\mathbb{E}_t\Pi_t^{(B)}||}$. This implies that if the transmission of monetary policy in a country becomes more efficient at any given time, then it is optimal for the central bank to exert more effort into stabilizing the rate of inflation in that country.

3.2 Structural model examples

To illustrate how the test statistic can correct mistakes made in the policy making process this section presents a simple stylized New Keynesian model of a monetary union with a central bank that conducts monetary policy by setting the short term interest rate. These examples serve to illustrate how the test statistic would work when the data is generated by the New Keynesian model and when the model is known to the researcher. We emphasize that this section is intended for clarification and that the main contribution of the paper is to provide a method of testing the optimality of policy choices of a monetary union's central bank when the true values of the parameters and functional forms of the underlying economic model is unknown. Furthermore, in order to facilitate the comparison to the efficient solution of the model, in this section we will use the social welfare loss function rather than a loss function specified by a mandate.

We consider the two-country optimizing model with sticky prices of Benigno (2004). The model reflects a monetary union that is made up of two regions, H and F, a single central bank, and two fiscal authorities. To sharpen the discussion, we will consider the case where prices are sticky in region H, while they are fully flexible in region F. In such a setting the central bank should not put any weight on the rate of inflation in the flexible price region but focus solely on the rate of inflation in the sticky price region.

The log-linear equilibrium fluctuations of the model under this assumption are captured by an intertemporal IS equation and a New Keynesian Phillips curve. The demand side of the model is represented by the IS curve

$$y_t^W = \mathbb{E}_t(y_{t+1}^W) - \frac{1}{\sigma}(i_t - \mathbb{E}_t(\pi_{t+1}^W) - r_t^e)$$

where y_t^W is the output gap of the monetary union, π_t^W is the union level rate of inflation, i_t is the nominal interest rate set by the central bank, and r_t^e is the natural rate of interest.

The supply block of the model consists of a New Keynesian Phillips curve for region H

$$\pi_t^H = \kappa y_t^W + \beta \mathbb{E}_t(\pi_{t+1}^H) + \varepsilon_t^s$$

where κ is a function of the structural parameters of the model and ε_t^s is an i.i.d. cost-push shock.

The second-order approximation of the welfare loss function in this case is

$$W = E_0 \sum_{t=0}^{+\infty} \beta^t (\lambda (y_t^W)^2 + (\pi_t^H)^2)$$

where λ is a function of the structural parameters of the model.

As shown in Benigno (2004), in the case where one region has flexible prices while the other has sticky prices, it is optimal to stabilize inflation in the region with sticky prices and under optimal policy, the efficient allocation is reached. The central bank can achieve the efficient allocation by following the Taylor rule specified by

$$i_t = r_t^e + \phi \pi_t^H$$

where $\phi_\pi = \frac{\kappa\sigma}{\lambda}$ which we assume to be greater than unity to ensure determinacy of the solution. Furthermore, the value of inflation that is consistent with this policy choice is $\pi_t^{H\star} = \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s$.

To show how the test statistic can both detect optimization failures in the conduct of monetary policy, discretionary or systematic, and be used to correct the policy choice to achieve optimal policy, we assume that the central bank either adds an erroneous discretionary shock to the interest rate or follows a non-optimal Taylor rule when responding to a non-zero cost-push shock. A comparable analysis applies when the central bank makes mistakes about the value of the model variables or other model parameters but here, we restrict the discussion to these two cases for expositional purposes.

The first situation we consider is a central bank that correctly realizes that it should only react to the rate of inflation in region H but is mistaken by the true value of ϕ. The second situation we consider is a central bank that is correct about the value of ϕ, but incorrectly adds a discretionary shock to the interest rate. In the third situation, the central bank that again knows the correct value of ϕ but also reacts to inflation in the flexible price region F. The final situation considered is a combination of the previous two. The central bank incorrectly targets a weighted average of the rates of inflation in regions H and F, that is, the central bank incorrectly puts a positive weight on the rate of inflation in region F and as a result responds incorrectly to the rate of inflation in region H. All three situations could arise, e.g., because of biased estimates of the underlying structural parameters.

In all cases, the causal effects of a unit change in i_t are given by

$$\mathcal{R} = [-\kappa\sigma^{-1}, 0, \ldots, 0, -\sqrt{\lambda}\sigma^{-1}, 0, \ldots, 0]'$$

Incorrect value of ϕ

We start with a central bank that is mistaken about the true value of the parameter ϕ in the Taylor rule and instead follows the non-optimal Taylor Rule

$$i_t = r_t^e + \tilde{\phi}\pi_t^H, \quad \tilde{\phi} = \phi(1+\gamma)$$

Call the prescription of this policy i_t^0. Under i_t^0, we have that $\mathbb{E}_t Y_t^0$ is given by

$$\mathbb{E}_t Y_t^0 = [\pi_t^{H,0}, 0, \ldots, 0, \sqrt{\lambda} y_t^{W0}, 0, \ldots, 0]'$$

where

$$\begin{aligned} \pi_t^{H,0} &= \frac{\lambda}{\lambda + \kappa^2(1+\gamma)} \varepsilon_t^s \\ y_t^{W0} &= -\frac{\phi(1+\gamma)}{\sigma} \frac{\lambda}{\lambda + \kappa^2(1+\gamma)} \varepsilon_t^s \end{aligned}$$

Thus we can compute the evaluation statistic as

$$\delta_t^\star = -(\mathcal{R}'\mathcal{R})^{-1}\mathcal{R}'\mathbb{E}_t Y_t^0 = -\left(\frac{\sigma\kappa\gamma}{\kappa^2 + \lambda}\right)\left(\frac{\lambda}{\lambda + \kappa^2(1+\gamma)}\right)\varepsilon_t^s$$

The first thing to note is that, since we are considering the response to a non-zero cost-push shock, we can reject the idea that the policy choice is optimal since $\delta_t^\star \neq 0$ which will be the case unless $\gamma = 0$, i.e., the central bank would be following the optimal Taylor rule. Secondly, to see how the test statistic can correct the policy i_t^0, consider

$$\begin{aligned} i_t &= i_t^0 + \delta_t^\star \\ &= r_t^e + \frac{\kappa\sigma}{\lambda} \frac{\lambda}{\lambda + \kappa^2} \varepsilon_t^s \\ &= r_t^e + \phi \pi_t^{H\star} \end{aligned}$$

where $\pi_t^{H\star} = \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s$ is the value of inflation in region H that is consistent with the policy i_t. Thus, as expected in a linear model, the test statistic corrects the misspecified Taylor rule and adjusts the policy choice to correspond to the prescription of the optimal Taylor rule.

Incorrectly adds discretionary shock

Consider a central bank that knows the true value of the parameter ϕ in the Taylor rule but incorrectly adds a discretionary shock to the interest rate

$$i_t = r_t^e + \phi\pi_t^H + \varepsilon_t^i$$

Reusing notation, call the prescription of this policy i_t^0. Under i_t^0, we have that $\mathbb{E}_t Y_t^0$ is given by

$$\mathbb{E}_t Y_t^0 = [\pi_t^{H,0}, 0, \ldots, 0, \sqrt{\lambda} y_t^{W0}, 0, \ldots, 0]'$$

where

$$\begin{aligned} \pi_t^{H,0} &= \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s - \frac{\kappa}{\sigma}\frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^i \\ y_t^{W0} &= -\frac{\phi}{\sigma}\frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s + \frac{\phi\kappa}{\sigma^2}\frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^i - \frac{1}{\sigma}\varepsilon_t^i \end{aligned}$$

Thus we can compute the evaluation statistic as

$$\delta_t^\star = -(\mathcal{R}'\mathcal{R})^{-1}\mathcal{R}'\mathbb{E}_t Y_t^0 = -\left(\frac{\lambda}{\lambda+\kappa^2}\right)\varepsilon_t^i$$

The first thing to note is that, since we are considering the response to a non-zero cost-push shock, we can reject the idea that the policy choice is optimal since $\delta_t^\star \neq 0$ which will be the case unless $\varepsilon_t^i = 0$, i.e., the central bank would be following the optimal Taylor rule. Secondly, to see how the test statistic can correct the policy i_t^0, consider

$$\begin{aligned} i_t &= i_t^0 + \delta_t^\star \\ &= r_t^e + \phi\frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s \\ &= r_t^e + \phi\pi_t^{H\star} \end{aligned}$$

where $\pi_t^{H\star} = \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s$ is the value of inflation in region H that is consistent with the policy i_t. Thus, as expected in a linear model, the test statistic corrects the misspecified Taylor rule and adjusts the policy choice to correspond to the prescription of the optimal Taylor rule.

Incorrectly reacting to π_t^F

Moving to the situation where the central bank knows the correct value of ϕ but incorrectly also reacts to the rate of inflation in region F, we have that the central bank follows a Taylor rule of the form

$$i_t = r_t^e + \phi\pi_t^H + \xi\pi_t^F$$

Reusing notation, call the prescription of this policy i_t^0. Under i_t^0, we have that $\mathbb{E}_t Y_t^0$ is again given by

$$\mathbb{E}_t Y_t^0 = [\pi_t^{H,0}, 0, \ldots, 0, \sqrt{\lambda} y_t^{W0}, 0, \ldots, 0]'$$

but now

$$\begin{aligned}
\pi_t^{H,0} &= \left(\frac{\lambda}{\lambda+\kappa^2}\right)\left(\frac{-\kappa\xi}{\sigma}\right)\pi_t^F + \left(\frac{\lambda}{\lambda+\kappa^2}\right)\varepsilon_t^s \\
y_t^{W0} &= -\left(\frac{\phi}{\sigma}\right)\left(\frac{\lambda}{\lambda+\kappa^2}\right)\left(\frac{-\kappa\xi}{\sigma}\right)\pi_t^F - \frac{\phi}{\sigma}\left(\frac{\lambda}{\lambda+\kappa^2}\right)\varepsilon_t^s - \frac{\xi}{\sigma}\pi_t^F
\end{aligned}$$

The calculation of the evaluation statistic yields

$$\delta_t^\star = -(\mathcal{R}'\mathcal{R})^{-1}\mathcal{R}'\mathbb{E}_t Y_t^0 = -\frac{\lambda\xi}{\kappa^2+\lambda}\pi_t^F$$

As in the previous example, we can reject the idea that the central bank sets the interest rate optimally since as long as $\xi \neq 0$ we have that $\delta_t^\star \neq 0$. Furthermore, consider using the test statistic to correct the policy choice

$$\begin{aligned}
i_t &= i_t^0 + \delta_t^\star \\
&= r_t^e + \phi\pi_t^H + \xi\pi_t^F - \frac{\lambda\xi}{\kappa^2+\lambda}\pi_t^F \\
&= r_t^e + \phi\frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s \\
&= r_t^e + \phi\pi_t^{H\star}
\end{aligned}$$

where $\pi_t^{H\star} = \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s$ is the value of inflation in region H that is consistent with the policy i_t. Thus, again, using the test statistic additively with the initial policy choice corrects the misspecified policy prescription to correspond with the optimal choice of policy response.

Incorrectly reacting to weighted average of inflation

Finally, we consider a combination of the previous two cases where the central bank incorrectly targets a weighted average of the per country rates of inflation. The resulting Taylor rule takes the form

$$i_t = r_t^e + \phi((1-\gamma)\pi_t^H + \gamma\pi_t^F)$$

Reusing notation once more, call the prescription of this policy i_t^0. Under i_t^0, we have that $\mathbb{E}_t Y_t^0$ is again given by

$$\mathbb{E}_t Y_t^0 = [\pi_t^{H,0}, 0, \ldots, 0, \sqrt{\lambda}y_t^{W0}, 0, \ldots, 0]'$$

but in this case

$$\begin{aligned}
\pi_t^{H,0} &= \left(\frac{\lambda}{\lambda+\kappa^2(1-\gamma)}\right)\left(\frac{-\kappa\phi\gamma}{\sigma}\right)\pi_t^F + \left(\frac{\lambda}{\lambda+\kappa^2(1-\gamma)}\right)\varepsilon_t^s \\
y_t^{W0} &= -\left(\frac{\phi(1-\gamma)}{\sigma}\right)\left(\frac{\lambda}{\lambda+\kappa^2(1-\gamma)}\right)\left(\frac{-\kappa\phi\gamma}{\sigma}\right)\pi_t^F - \frac{\phi\gamma}{\sigma}\pi_t^F \\
&\quad - \left(\frac{\phi(1-\gamma)}{\sigma}\right)\left(\frac{\lambda}{\lambda+\kappa^2(1-\gamma)}\right)\varepsilon_t^s
\end{aligned}$$

Thus, we can compute the evaluation statistic as

$$\begin{aligned}
\delta_t^\star &= -(\mathcal{R}'\mathcal{R})^{-1}\mathcal{R}'\mathbb{E}_t Y_t^0 \\
&= -\left(\frac{\sigma}{\kappa^2+\lambda}\right)\left(\left(\frac{\lambda}{\lambda+\kappa^2(1-\gamma)}\right)\left(\frac{-\kappa\phi\gamma}{\sigma}\right)(-\gamma\kappa)+\kappa\gamma\right)\pi_t^F - \frac{\lambda\kappa\gamma}{\lambda+\kappa^2(1-\gamma)}\varepsilon_t^s
\end{aligned}$$

Once again, we can reject the notion that the central bank is choosing its policy optimality as long as $\gamma \neq 0$. Once more, consider adjusting the initial policy choice by the evaluation statistic

$$\begin{aligned}
i_t &= i_t^0 + \delta_t^\star \\
&= r_t^e + \phi(1-\gamma)\pi_t^H + \phi\gamma\pi_t^F \\
&\quad - \left(\frac{\sigma}{\kappa^2+\lambda}\right)\left(\left(\frac{\lambda}{\lambda+\kappa^2(1-\gamma)}\right)\left(\frac{-\kappa\phi\gamma}{\sigma}\right)(-\gamma\kappa)+\kappa\gamma\right)\pi_t^F - \frac{\lambda\kappa\gamma}{\lambda+\kappa^2(1-\gamma)}\varepsilon_t^s \\
&= r_t^e + \phi\frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s \\
&= r_t^e + \phi\pi_t^{H\star}
\end{aligned}$$

where $\pi_t^{H\star} = \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t^s$ is the value of inflation in region H that is consistent with the policy i_t. Thus, once again, the test statistic corrects the initial incorrect policy choice to correspond with the optimal choice for the policy instruments.

4 Implementing the test for the optimality of ECB policy decisions

Let us now turn to the concrete example of the European Central Bank. The ECB has a clear and narrow mandate of maintaining Euro area inflation below, but close to, 2% over the medium term. At the time of writing there are $N = 19$ euro area member countries, Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia, and Spain.[4] We thus need causal estimates of the effect of ECB monetary policy on the inflation rate in all 19 member countries for the three policy tools we consider along with the ECB's inflation forecasts for each country.

4.1 Forecasts

We use the forecasts published by the ECB concurrently with its monetary policy decision dates as approximations to the ECB's expectations of the rate of inflation in each member country conditional on its current policy decision. The ECB has published member country level inflation forecasts semi-annually since the second quarter of 2016 with the latest forecast available being from the second quarter of 2020 giving a total of 9 monetary policy decisions that can be evaluated. The forecasts are at the annual level and represent a forecast horizon of 2.5 to 3 years depending on the time of year the forecast is published. This implies that the horizon over which the evaluation can be performed is 11 or 13 quarters, including the quarter the forecast is made, depending on the forecasting round. To be able to use the forecasts with quarterly impulse responses of the rates of inflation to monetary shocks, we obtain quarterly forecasts by temporally disaggregating the implied forecasted annual values of each country's HICP.

The limited number of forecasts available presents a problem when it comes to the estimation of the mean squared forecast error. In fact, the data only offers 4 forecasting rounds that can be fully evaluated in terms of forecast errors. Therefore we will use estimates of the root mean squared forecast error for the Eurosystem/ECB's forecast as estimated in Kontogeorgos and Lambrias (2019). In the paper they present values for 1, 4, and 8 quarters ahead. We fit a logarithmic curve through these values in line with theory and evidence that the forecast error increases along the forecast horizon. We use the euro area aggregate numbers for each country.

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[4] Since the analysis was performed Croatia has adopted the Euro in January of 2023.

4.2 Causal Effects

While the local projection methods used to estimate the causal effects are standard, we include a brief discussion to be explicit about all the necessary steps. Firstly, as is standard in the treatment-effect and sufficient statistics literature the present paper will assume a constant policy regime over the period under analysis. Secondly, to address any potential endogeneity problems we will use instrumental variables to estimate the causal effects. Note that, in the present paper, the matrix of causal effects $\mathcal{R}^0$ consists of 57 blocks, the responses of the target variable in each of the 19 countries to the three policy tools.

$$\mathcal{R}^0 = \begin{pmatrix} \mathcal{R}^0_{1,TR} & \mathcal{R}^0_{1,FG} & \mathcal{R}^0_{1,QE} \\ & \vdots & \\ \mathcal{R}^0_{19,TR} & \mathcal{R}^0_{19,FG} & \mathcal{R}^0_{19,QE} \end{pmatrix} \tag{21}$$

where $\mathcal{R}^0_{i,j} = (r_{ij,0}, r_{ij,1} \ldots)'$ and $r_{ij,h}$ is the causal effect of policy tool j on the target variable in country i at horizon h.

Additionally, note that equation (7) takes a form very reminiscent of local projections. Beginning with the standard local projection (ignoring constants and other controls or assuming they have already been projected out), we can estimate the elements of these blocks as the sequences $r_{ij,h}, h = 0, \ldots, H$ where H is the maximum horizon from the sequence of regressions

$$y_{i,t+h} = r_{ij,h} v_{j,t} + u_{ij,h,t+h} \tag{22}$$

for target variable i and where $v_{j,t}$ is the fitted value from a first stage regression of policy tool j on an external instrument and $u_{ij,h,t+h}$ is the combination of the corresponding element of Υ_t and the other policy news shock.

The instruments used in the estimation are constructed as in the recent paper of Altavilla et al. (2019) where they create a database containing intra-day asset price changes around monetary policy decisions of the ECB. They use the database to construct proxies for the Target Rate, Forward Guidance, and Quantitative Easing surprises, the first two following the methodology of Gürkaynak (2005) and Gürkaynak, Sack and Swanson (2005) which build on Kuttner (2001) and the last following the work of Swanson (2017). The proxies correspond to factors estimated from changes in the yields of Euro area risk-free rates with maturities from one month up to ten years within a specified window of time around the press release of the monetary policy decision and the following press-conference.

While there are many studies that examine the financial and real effects of ECB policy as measured using high frequency surprises (see, e.g., Corsetti, Duarte and Mann, 2020, Jarocinski and Karadi, 2020, Andrada and Ferroni, 2020, Schrimpf, 2019, or Brand, Buncic and Turunen, 2010) there are, however, to the best of the author's knowledge, none that examine the effect on inflation of changes in all three policy tools, the Target Rate, Forward Guidance, and Quantitative Easing, simultaneously for all euro area member countries.

The causal effects are estimated jointly over the horizon for all member countries and policy tools using instrumental variable regression following a rewriting of the model as in Barnichon and Brownlees (2019). Starting from the model in (22), let s denote the estimation sample where $s = 1, \dots, T$. Let $\mathcal{Y}_{s,i} = (y_{s,i}, \dots, y_{\min(T,s+H),i})'$ whose dimension is denoted by d_s. Let $\mathcal{X}_{r,s,i,j}$ be a $(H + 1 \times H + 1)$ diagonal matrix with the diagonal elements equal to $v_{s,j}$. Now let $\mathcal{Y}_i = (\mathcal{Y}'_{1,i}, \dots, \mathcal{Y}'_{\min(T,s+H),i})'$ and $\mathcal{X}_{r,i,j} = (\mathcal{X}'_{r,1,i,j}, \dots, \mathcal{X}'_{r,\min(T,s+H),i,j})'$.

We can then write the regression model as

$$\mathcal{Y}_i = \mathcal{X}_{r,i,j}\mathcal{R}^{0,H}_{ij} + \mathcal{U} \tag{23}$$

where $\mathcal{R}^{0,H}_{i,j} = (r_{ij,0}, r_{ij,1} \dots, r_{ij,H})'$ and $\mathcal{U}$ denotes an error term. This estimator is asymptotically normally distributed under standard assumptions. We compute the variance matrix of the estimated impulse responses using Newey and West (1994). The monetary policy surprises used as instruments in the estimation are only available from 2002. Therefore, the sample period used for the estimation of the causal effects covers 2002Q1 to 2019Q4 to avoid including the period of the Covid-19 crisis that began early 2020. The estimation results are shown in Figures 1 to 4.

4.3 OPP

With the quarterly forecasts in hand, we convert them into deviations from the ECB's inflation target. Stacking the country level forecasts at each forecasting round gives the mean of the approximating distribution of the ECB's expectations of the path taken by the target variables conditional on its current policy choice, $\hat{Y}_{t|t}$. The covariance matrix of the approximating distribution is a block diagonal matrix where each block corresponds to a country and equals the root mean squared error from Kontogeorgos and Lambrias (2019). Given these two parameter estimates we can sample from the approximate distribution of $\mathbb{E}_t Y_t^0$.

Given estimates of the impulse response functions $\mathcal{R}_{i,j}^{0,H}$ for each country and policy instrument and their associated covariance matrices we can sample each of them from their approximating distributions. Stacking the sampled impulse response functions such that the columns correspond to the different policy instruments and each $H+1$ block of rows corresponds to the same country as in $\hat{Y}_{t|t}$ gives a draw of $\mathcal{R}$.

The final ingredient needed to calculate the OPP is the weighting matrix $\Omega = \text{diag}(\omega \otimes (\nu \otimes \beta))$. Since the ECB only has a single mandate, we have that $\nu = 1$. We further assume that $\beta_h = 1$ for $h = 0, \ldots, H$. This simplifies the weighting matrix to $\Omega = \text{diag}(\omega \otimes \iota)$ where ι is a $(H+1) \times 1$ vector of ones. Furthermore, the weights ω_i are selected to correspond to the countries shares of overall euro area household final monetary consumption expenditure. This is consistent with the weights each country gets in the composition of aggregate euro area HICP. Additionally, this is consistent with the weight inflation in each country gets in the social loss function in the structural analysis in Benigno (2004) under the assumption of equal nominal rigidities in all countries. With all the building blocks in place, we sample the conditional expectations and the causal effects from their approximating distributions 5000 times and calculate the test statistic in each case. We reject the hypothesis that a policy instrument was used optimally if zero lies outside the 16th or 84th quantiles of the resulting distribution, i.e., if the 68% confidence bounds do not include zero.

5 Empirical Results

5.1 Main OPP results

In this section we turn to the results of the test of whether the ECB's monetary policy decisions deviate from optimality. As previously mentioned, there are 9 monetary policy decisions available to be tested, from 2016Q2 to 2020Q2. In the available time frame, the ECB has experienced a period of negative short term interest rates. While there is some debate whether the zero lower bound has been binding for the ECB during this period of time or at what point below zero it does bind, we nonetheless include the Target Rate instrument in the analysis. Performing the analysis under the assumption that the zero lower bound binds, and therefore excluding the Target Rate from the evaluation, does not change the qualitative results for the Forward Guidance and Quantitative Easing tools and quantitatively the difference is marginal with the largest difference amounting to 3 basis points. The following discussion thus remains qualitatively the same whether the zero lower bound presented a binding constraint for the ECB or not.

Figure 5 presents the median along with the 16th and 84th percentiles of the approximate distribution of $\delta_t^\star$ calculated according to section 2.3.[5] The first panel of the figure presents the results for the Target Rate instrument. The results show that we cannot reject the hypothesis that the Target Rate was optimally set. In addition, we see that the value of the OPP statistic is quite small, ranging between -4 basis points to 8 basis points. The fact that there is no suboptimality in the setting of the Target Rate tool, combined with the marginally small negative values of the OPP statistic, when it is negative, can be interpreted as evidence that the zero lower bound was in fact not binding for the ECB during the period under evaluation. Had the zero lower bound been binding, the OPP statistic would need to be negative and statistically significant since for the zero lower bound to bind, the optimal policy decision would have to be to lower the interest rate in the presence of the impossibility of lowering the interest rate.

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[5] For completeness, Figure 6 presents the results assuming the zero lower bound binds.

The middle panel of Figure 5 shows the results for the Forward Guidance instrument. Over the period 2016Q2 to 2020Q2 we can reject the hypothesis that the ECB used its Forward Guidance tool optimally 6 out of 9 instances. Only in 2017Q2, 2018Q2, and 2018Q4 can we not reject optimality. When we can reject optimality, the OPP indicates that the ECB should have used its Forward Guidance more aggressively to lower the slope of the yield curve by 7 to 25 basis points, thus accommodating the economy more than it did in actuality.

Finally, the bottom panel of Figure 5 shows the results for the Quantitative Easing tool. The figure shows that we can reject the hypothesis that the ECB used its Quantitative Easing tool optimality in all cases. In all cases, the OPP indicates that the ECB should have steered the long term interest rate such that it was about 8-38 basis points lower than it was in actuality and thus, as for the Forward Guidance, stimulate the economy further.

5.2 What drives the results

To examine possible heterogeneities across member countries in driving the results of the test, Figures 7 to 9 present the contribution of member countries, calculated as $W_i\delta_{it}$, to the union level test statistic δ_t. For ease of legibility, the figure presents the contribution of the five largest economies, Germany, France, Italy, Spain, and the Netherlands, and the collective contribution of the rest of the member countries.

Figure 7 shows the results for the Target Rate instrument. An interesting observation that the figure reveals is that in 5 out of the 9 monetary policy decisions under consideration, the contribution of France, the second largest economy in the union, has an opposite sign to the union level statistic. This is never the case for Germany, the largest economy in the union, as the sign of the union level statistic is always the same as the sign of the statistic for Germany. The figure also reveals that in 4 out of the 9 decisions, the contribution of the smallest 14 member countries has the opposite sign to the union level statistic. Therefore, when it comes to the Target Rate policy instrument some heterogeneity across the member countries emerges.

Turning to the Forward Guidance instrument, Figure 8 presents the decomposition. As opposed to the Target Rate instrument, the sign of the union level statistic and the contribution of France always matches. For Germany however, the sign of the contribution is different from the union level statistic in 5 out of 9 instances. For Italy, the concordance is 7 out of 9 instances, and for Spain it is 8 out of 9 decisions. For the smallest 14 member countries the sign of their contribution is the same as the sign of the union level statistic in all but 1 instance. In fact, when the sign of the contribution of a country is opposite to the sign of the union level statistic, their contribution is usually very small numerically indicating that there does not seem to be much heterogeneity when it comes to the usage of the Forward Guidance tool.

Finally, Figure 9 presents the decomposition for the Quantitative Easing instrument. A very clear pattern is visible from the figure. The sign of the contributions of all member countries agrees with the sign of the union level statistic in all cases with the sole exception of the Netherlands in 2017Q4, where the contribution is very small. Thus, there is a great level of homogeneity with regards to the use of the Quantitative Easing instrument.

5.3 Country level results

It is important to keep in mind that not rejecting optimality at the union level does not imply that we cannot reject optimality for all member countries. Additionally, rejecting optimality at the union level does not imply that we reject optimality for all member countries. In this section we will explore the optimality of monetary policy in each country by itself for each policy instrument

5.3.1 The Target Rate Instrument

Figure 10 presents the country level test statistic for each country, along with the 16th and 84th percentiles of the approximate distribution of $\delta^{\star}_{it}$ calculated following the same steps as in the discussion in section 2.3. Furthermore, Table 1 collects all instances of rejection for each country and each policy instrument.

For the Target Rate tool there are a handful of instances where we can reject optimality in the use of the Target Rate at the country level. In 2016Q2 we can reject optimality for Finland and the Netherlands. In 2016Q4 we again reject optimality for both of them and additionally for Ireland but in 2017Q2 we can only reject optimality in the Netherlands. In all of these instances, the country level test statistics indicate that the Target Rate should have been higher in order for it to have been optimal in the previously mentioned countries. During 2017Q4 and 2018Q2 we can reject optimality in Finland and Estonia with the test statistics calling for a higher rate in Finland but a lower rate in Estonia. In 2018Q4 we can reject optimality in Ireland, in 2019Q2 in Estonia, and in 2019Q4 for Finland. The country level test statistic indicates that the Target Rate should have been lower in order to be optimal for Estonia, but higher in Ireland and Finland. Finally, in 2020Q2, we can reject optimality in all the countries mentioned above, Estonia, Finland, Ireland and the Netherlands, in addition to Austria. In all countries the test statistic calls for a higher interest rate.

In summary, some suboptimality in the setting of the Target Rate does exist across the member countries even though we can never reject optimality at the union level. This suboptimality is, however, isolated to a handful of member countries with Finland being worst off with 6 rejections of optimality out of the 9 decisions under review.

5.3.2 The Forward Guidance Instrument

For the Forward Guidance instrument we turn to Figure 11. Quite opposed to the results for the Target Rate we see that we can reject optimality at some point in every member country except for Austria. Furthermore, in Ireland we can reject optimality for every single decision and in Latvia all but one decision. When it comes to individual decisions, we see that in 2016Q2 we can reject optimality in 16 out of the 19 member countries, with Austria, Belgium and the Netherlands the only countries where we cannot reject optimality. In all cases the test statistic indicates that Forward Guidance should have been used to make the slope of the yield curve lower than it was. The following period, in 2016Q4, the number of countries where we can reject optimality is halved and we can only reject in the non-core countries of Cyprus, Ireland, Italy, Lithuania, Latvia, Malta, Slovakia, and Slovenia. In all cases the country level test statistics indicate that the slope of the yield curve would have had to be lower for the optimality condition not to be rejected in each country. Through the next few decisions, the number of countries where we can reject optimality trends down to 4 in 2018Q4 with the set of countries where we can reject from 2017Q2 to 2018Q4 consisting of the peripheral economies with the exception of Finland, where we can reject optimality in 2017Q2 and 2017Q4, the Netherlands in 2018Q2, and Belgium where we can reject optimality in 2018Q4. For Finland and the Netherlands, the test statistic indicates that the slope should have been lower for policy to be optimal in these countries individually, while in Belgium the test statistic calls for the slope to have been steeper. During this period, we can always reject optimality in Latvia, with the test statistic indicating that the slope should have been steeper for optimality to be reached. The same is true in Lithuania in 2017Q2 and 2017Q4, Luxembourg in 2018Q2 and Estonia in 2018Q4. On the other hand, we can reject optimality in Ireland over the same time period of 2017Q2 to 2018Q4, with the test statistic calling for a lower slope to achieve optimality. The same result is found in Finland, Malta and Slovakia in 2017Q2, Cyprus, Finland, Greece and Italy in 2017Q4, and in Greece and the Netherlands in 2018Q2. In 2019Q2, we can reject optimality in 6 of the member economies, with the test statistics calling for an increase in the slope in Latvia, the Netherlands and Slovakia, but in Cyprus, Ireland and Italy the test statistics call for the slope to be lower. At the following decision in 2019Q4, however, we can reject optimality in 5 member countries, Belgium, Cyprus, Ireland, Italy, and Portugal, and in all cases the country level test statistics call for the slope to have been lower in order to attain optimality. Finally, in 2020Q2, we can reject optimality in 11 out of 19 member economies with optimality not being rejected only in Austria, Germany, Spain, Estonia, France, Luxembourg, the Netherlands, and Slovakia. In all of the member countries where we can reject optimality, the test statistic calls for the slope to have been lower for optimality to be reached.

For the Forward Guidance instrument there is a substantial amount of heterogeneity in the signs of the adjustments necessary to restore optimality in the instances where we can reject. Furthermore, in the instances where we can reject optimality at the union level, there are cases where we can reject optimality at the country level where the sign of the optimal adjustment differs from the sign of the union level adjustment. In these instances, if the ECB would have adjusted its Forward Guidance policy to restore optimality at the union level, some member countries would be even worse off than with no adjustment at all. Another interesting pattern that emerges is that in all the cases where we can reject optimality at the union level, Italy is in the set of countries we can reject optimality at the country level with the sign of the optimal adjustment for Italy matching the sign of the optimal adjustment at the union level.

5.3.3 The Quantitative Easing Instrument

Finally, Figure 12 presents the country level results for the Quantitative Easing instrument. There are only 9 member countries in which we can reject the optimality of the use of the Quantitative Easing instrument at some point. These are Austria, Belgium, Cyprus, France, Greece, Ireland, Italy, Latvia, and Portugal. In all cases of rejection, the country level test statistics indicate that the Quantitative Easing instrument should have been used to bring the long-run interest rate lower than it actually was. In Portugal, we can, in fact, reject optimality at every single decision under consideration. In France and Ireland, we can reject optimality in all but 2 decisions in each country, in France from 2016Q2 to 2017Q4, and again from 2019Q2 to 2020Q2 and from 2017Q2-2020Q2 in Ireland. In Cyprus we can reject optimality in 2016Q2 and 2016Q4, and again from 2019Q2-2020Q2. In Greece we have a single rejection in 2017Q2 and in Italy we can reject optimality in 2017Q4, 2019Q4 and 2020Q2. In Latvia we can reject optimality in 2017Q4 and 2020Q2 and, finally, in Austria and Belgium we can reject optimality in 2019Q4 and 2020Q2.

An interesting observation is that in the set of countries where we can reject optimality, we have 8 out of the 10 countries with the highest public debt to GDP ratio in the Eurozone with Latvia being the outlier as the country with the 16th highest public debt to GDP ratio.

However, Latvia, along with Cyprus, Greece, Ireland, and Portugal, did receive a bailout during the European Sovereign Debt Crisis. This set of countries is therefore made up of countries that, in all likelihood, are more sensitive to changes in the long-run interest rate which would explain why the Quantitative Easing tool would be an effective instrument for conducting monetary policy.

5.4 The Covid crisis - 2020Q2

The monetary policy decision during 2020Q2 makes for an interesting case study for several reasons. The decision came at the onset of the Covid-19 pandemic and the ensuing economic crisis with its associated policy responses. Additionally, out of the decisions under review in the present paper, the 2020Q2 policy decision has the highest number of country level rejections across all policy instruments. Finally, it is the decision that has the largest deviation from optimality, as measured by the size of the test statistic, for the Quantitative Easing instrument and the second largest deviation for the Forward Guidance instrument. It is therefore of interest to delve into and understand the circumstances of the 2020Q2 monetary policy decision.

Compared to the inflation outlook at the previous decision in 2019Q4 in the member countries, the inflation forecasts were on average revised downwards over the forecast horizon. For the year 2020, this downward revision was up to 3 percentage points and a minimum of 0.5 percentage points with the mean downwards revision of 1.23 percentage points in the member countries. In 2021, the inflation forecasts were revised downwards, on average, by 0.75 percentage points with the largest downwards revision being 2 percentage points and the smallest downwards revision being 0.1 percentage points. The inflation forecast in Greece was, however, revised upward by 0.3 percentage points, the only upwards revision to the inflation forecasts for 2021. The forecast horizon ends in 2022 in which the member country inflation forecasts were revised downwards, on average, by 0.22 percentage points. For 2022, the inflation forecast was revised downwards in 14 out of the 19 member countries, with the largest downwards revision being 0.6 percentage points. In the 5 member countries for which the inflation forecast for 2022 was revised upwards the largest revision was by 0.4 percentage points. A clear pattern emerges in which the monetary authority perceived the shocks hitting the member economies as having the greatest impact in the current year and fading out over the next couple of years with the inflation outlook being lower than previously predicted for the vast majority of the member countries.

Between 2019Q4 and 2020Q2, the short run money market interest rate remained practically unchanged in the Euro area. The two year interest rate, which is associated with the Forward Guidance instrument in the present paper, rose by roughly 16 basis points leading to the slope of the yield curve to become steeper by 17 basis points between the two periods, effectively tightening monetary policy. A similar story is found for the 10 year interest rate, which is associated with the Quantitative Easing instrument, which increased by almost 20 basis points over the same period, again representing a tightening of monetary policy. It is therefore interesting to note that the prescription of the test statistic in 2020Q2 is to use the Forward Guidance instrument to lower the slope of the yield curve by 20 basis points, and the Quantitative Easing instrument to lower the 10 year interest rate by 38 basis points. The OPP therefore prescribes that the slope should have been kept unchanged between 2019Q4 and 2020Q2 while the increase in the 10 year interest rate should have been netted out and lowered by about 20 basis points more to stimulate the member economies and support recovery out of the Covid crisis.

Figure 13 shows the deviation from the inflation target of the forecasted paths of the rate of inflation for the five largest Euro area economies, along with two hypothetical paths for the rate of inflation based on two thought experiments. For all five economies, the forecasted paths share the common feature noted above that the deviation is the largest at the start of the forecast horizon and gradually decreases towards the end of the horizon. This reflects how the dominating shock to the economies at this point in time is common to all member economies which, to an extent, makes the proper course of action of the central bank more clear cut.

The figure also shows two thought experiments. The first asks what would the path of the rate of inflation be if the initial policy choice would be adjusted by the value of the test statistic for the union as a whole, $\delta_t^\star$, while the second asks what would be the path of the rate of inflation if the adjustment would be done by the test statistic for each country individually, $\delta_{it}^\star$. In both cases it is evident that these adjustments bring the deviation of path of the rate of inflation from its target closer to zero and that, in general, the adjustment according to the country specific test statistic is more efficient at doing so.

Based on these hypothetical paths for the rate of inflation we can evaluate the relative reduction in the country specific loss achieved by adjusting the initial choice of monetary policy by the optimal adjustment. This is done in Table 2. For the union level adjustment, we see that the per country loss could be lowered by between 5 and 55%, with the average reduction in the five largest economies at 37%. These figures show that the ECB could have conducted its monetary policy more optimally and that the gain from doing so is sizable.

Had the countries had independent monetary policies, however, and the national central banks been able to adjust by the country level test statistic we see that the loss could have been lowered even further. For the five largest economies, the loss would have been between 57-91% lower with an average reduction of 74%. This finding highlights the potentially significant country level cost of forfeiting their independent monetary policies when joining a monetary union, a topic we turn to in the next section.

5.5 Cost of Joining the Union

The thought experiment in the previous section can be expanded upon in the following way. For each country and at each time period we can calculate the expected country specific losses given no adjustment to the ECB's chosen policy, the expected losses given the optimal adjustment according to the union level test statistic, and the expected losses given the optimal adjustments according to the country level test statistics. The underlying assumption is that if the country were not a member of the monetary union, its loss function is a country specific version of the union level loss function. That is, only a function of the deviation of inflation from the inflation target, which again is assumed to equal the union level inflation target. The first row of both panels of Table 3 shows the median per country expected loss after adjusting for the union level statistic relative to the no-adjustment expected loss. The median reduction in per country expected loss is 33% with the largest reduction at 69% for Greece. However, the results reveal that correcting monetary policy according to the union level test statistic does not lead to a reduction in the expected loss for every member country. In fact, the median expected loss is actually increased when policy is adjusted to the union level statistic in Estonia, Lithuania, Latvia and Slovakia with their median losses being 1.5, 1.57, 2.11, and 1.94 times their expected loss without the union level optimal policy adjustment. This increase in the expected loss could happen, for example, if the business cycles in these economies are asynchronous to the business cycles in the other member countries. In that case, the idiosyncratic economic developments in the asynchronous member countries can call for vastly different policy responses than in the more synchronous member country economies. This seems to be the case here as in the meta-analysis of Campos, Fidrmuc and Korhonen (2019), who studies business cycle synchronization in 16 Euro area countries, they find that Estonia, Lithuania, Latvia, and Slovakia are in the five least synchronized Euro area economies. Additionally, they find that the level of business cycle synchronization in these countries has not changed with the adoption of the Euro, whereas in the other Euro area economies, business cycle synchronization has increased.

The second row of both panels in Table 3 shows the median expected loss in each country after adjusting monetary policy by the country specific test statistics, relative to the expected loss of no-adjustment. As would be expected, the expected loss after adjusting by the country level test statistics is in all cases lower than the expected loss after adjusting by the union level test statistic and in all cases, it is lower than the expected loss of not adjusting policy. The median reduction in the expected loss is 77% and ranges between 44% in Luxembourg, up to 95% in Finland. We can therefore conclude that all of the member countries pay a cost when giving up their independent monetary policy in the sense that their expected losses under optimal policy in each country is always lower than the expected losses under optimal policy at the union level.

The bottom row of both panels in Table 3 quantifies the cost of joining the Euro area for each country as the expected loss under optimal policy in each country relative to the expected loss under optimal policy at the union level. We find that the median reduction in expected loss from having independent monetary policy is 58% with the smallest reduction in Spain at 4% but the largest at 92% in Finland. The asynchronous economies of Estonia, Lithuania, Latvia and Slovakia could lower their expected loss by 85-91% by conducting their monetary policies independently from the ECB. In fact, 12 out of the 19 member countries would have expected losses from country specific optimal policy that is more than 50% lower than the expected losses from the union level optimal policy, with Cyprus being on the border at 48% reduction. A further observation is that, unsurprisingly, the highest price of Euro membership is paid by countries on the periphery of the Euro area.

6 Conclusions

In this paper we present a framework to evaluate the optimality of monetary policy decisions made by a central bank in a monetary union. This tool determines whether the central bank's policy decisions are optimal, in the sense of being best suited to achieve the central bank's mandate and under linearity assumptions informs the central bank by how much it needs to adjust each policy tool in order to comply with the optimal choice. Furthermore, the tool does not rely on a specific structural model of the economy and thus aims to be robust to practical situations where the true structure of the economy is either unknown or too complex to put to paper.

We show, in the special case of a two country monetary union that the tool boils down to balancing average future inflation between the two countries, corrected for their relative weights in the union, the central bank's ability to transform a unit of inflation in one country into inflation in the other, and taking into account the dynamics of the transmission of monetary policy in both countries. Furthermore, to illustrate the workings of the test statistic we present a stylized New Keynesian model of a two country monetary union with a central bank conducting monetary policy using the short term interest rate. Working through several versions of missspecified Taylor rules for the central bank we show what one needs to calculate if the model were known to the researcher and the data truly is generated by the model. The examples show how the test statistic detects the central bank's failure to conduct policy optimally and how it can be used to adjust policy back to the optimum.

The conclusions from a retrospective analysis of the ECB's monetary policy decisions are that there are several instances of suboptimality in monetary policy decisions, both at the union level and at the individual country level. At the union level, we find no evidence that the Target Rate instrument was suboptimally used but for both the Forward Guidance and Quantitative Easing instruments we find suboptimality and that the ECB should have conducted its monetary policy such that both the slope and the long end of the yield curve would have been lower than it actually was.

We also find signs of heterogeneity in the optimal prescriptions for the individual member countries with respect to the union level prescription for the Target Rate. While the sign of the union level statistic always matches the test statistic for Germany, the signs are different in about half the cases when it comes to the contribution of the 14 smallest union members. For the Forward Guidance instrument we find some heterogeneity in the policy prescriptions in two instances. Interestingly, the sign for Germany differs from the sign for the union in about half of the cases. For the Quantitative Easing instrument, there is only one instance of one of the five largest economies having a test statistic with a different sign than the union level statistic, indicating substantial homogeneity when it comes to the prescriptions for that instrument.

In terms of optimization failures at the country level, we find that we can reject optimality in 7 to 24 instances per period for the three policy instruments and the 19 economies. The least frequent are for the Target Rate instrument with 1-5 rejections per period, the second is the Quantitative Easing instrument at 2-8 rejections per period and finally the Forward Guidance instrument at 4-16 rejections per period. We find that the cost of giving up independent monetary policy can be very substantial. Were the member countries to conduct their monetary policy independently and optimally, the median reduction in expected loss is 58% relative to the expected loss of being in a monetary union that conducts its monetary policy optimally. The highest price is paid by Finland which could expect 92% lower expected loss if it conducted its optimal policy independently. On the other hand, the lowest price is paid by Spain at 4%. Estonia, Lithuania, Latvia and Slovakia have business cycles that are not well synchronized with the rest of the Euro area business cycles. This leads to the result that if the ECB had adjusted its policy according to the prescription of the optimal adjustment, these countries would have been worse off than had no adjustment been made. The result of this is that they are among the countries paying the highest price of membership, assuming that the country level loss function mirrors the union level loss function, and could expect a 85-91% reduction in expected loss with independent monetary policies. Finally, given this assumption on the country level loss function, 12 out of the 19 member economies could reduce their expected loss by more than 50% by conducting monetary policy independently. The results further indicate that the highest price of Euro membership is paid by the peripheral economies.

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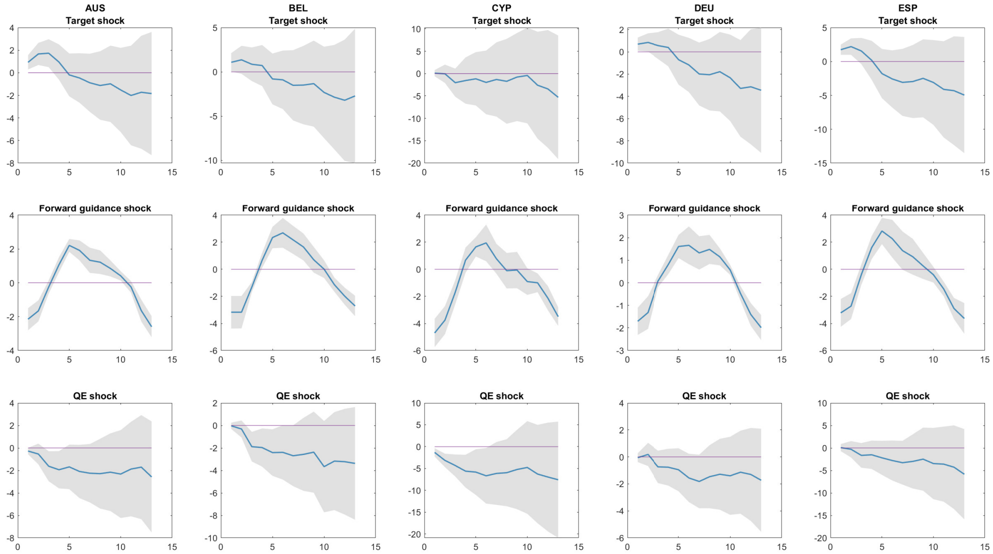
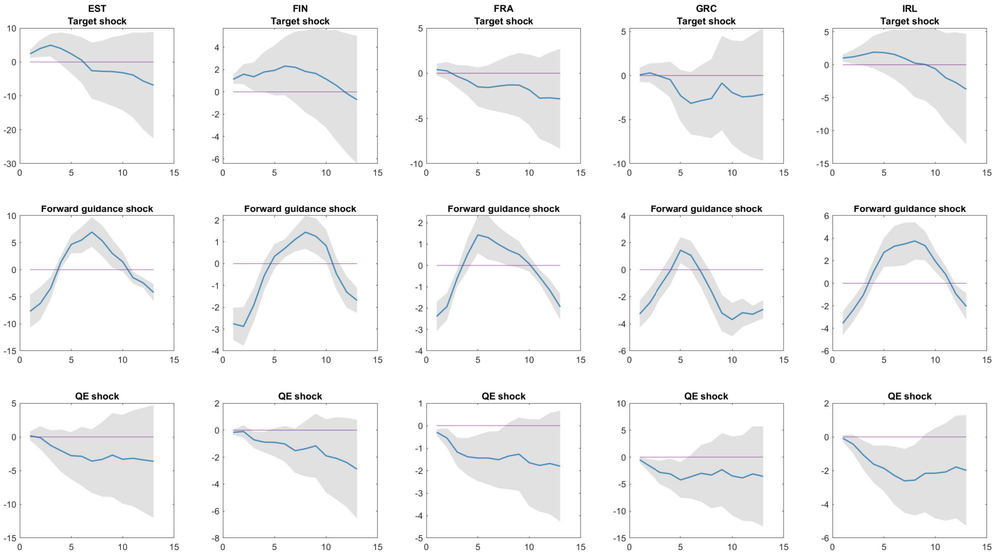
Figure 1: Causal Effect Estimates - AUS to ESP



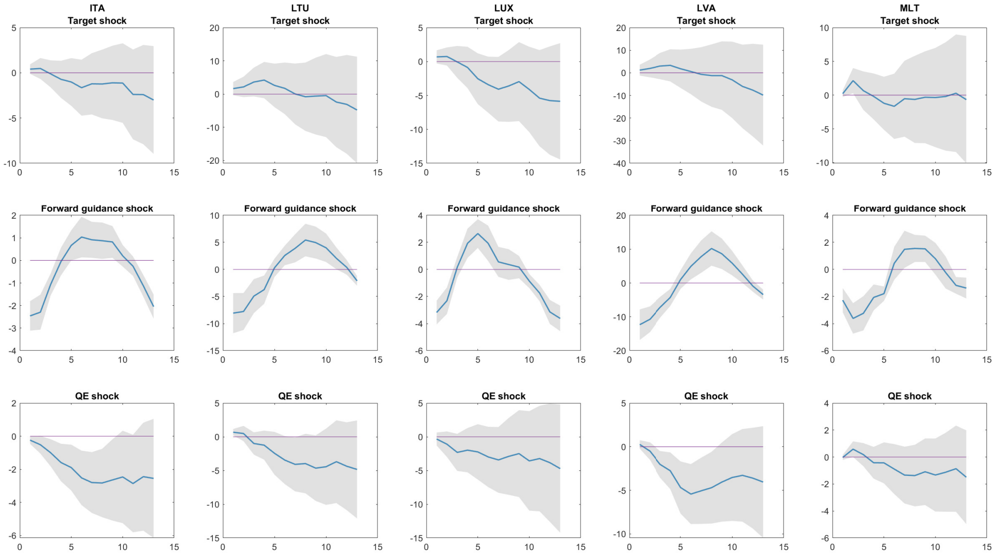
Notes: Impulse responses of country level inflation to a unit monetary policy shock in Austria, Belgium, Cyprus, Germany, and Spain. Top row: Response to a Target Rate shock. Middle row: Response to a Forward Guidance shock. Bottom row: Response to a Quantitative Easing shock. The shaded bands denote the 68 percent confidence intervals.

Figure 2: Causal Effect Estimates - EST to IRL



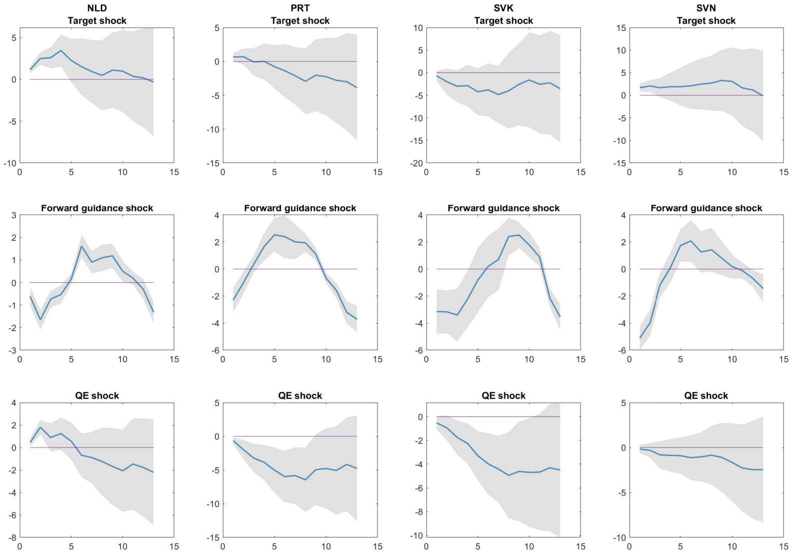
Notes: Impulse responses of country level inflation to a unit monetary policy shock in Estonia, Finland, France, Greece, and Ireland. Top row: Response to a Target Rate shock. Middle row: Response to a Forward Guidance shock. Bottom row: Response to a Quantitative Easing shock. The shaded bands denote the 68 percent confidence intervals.

Figure 3: Causal Effect Estimates - ITA to MLT



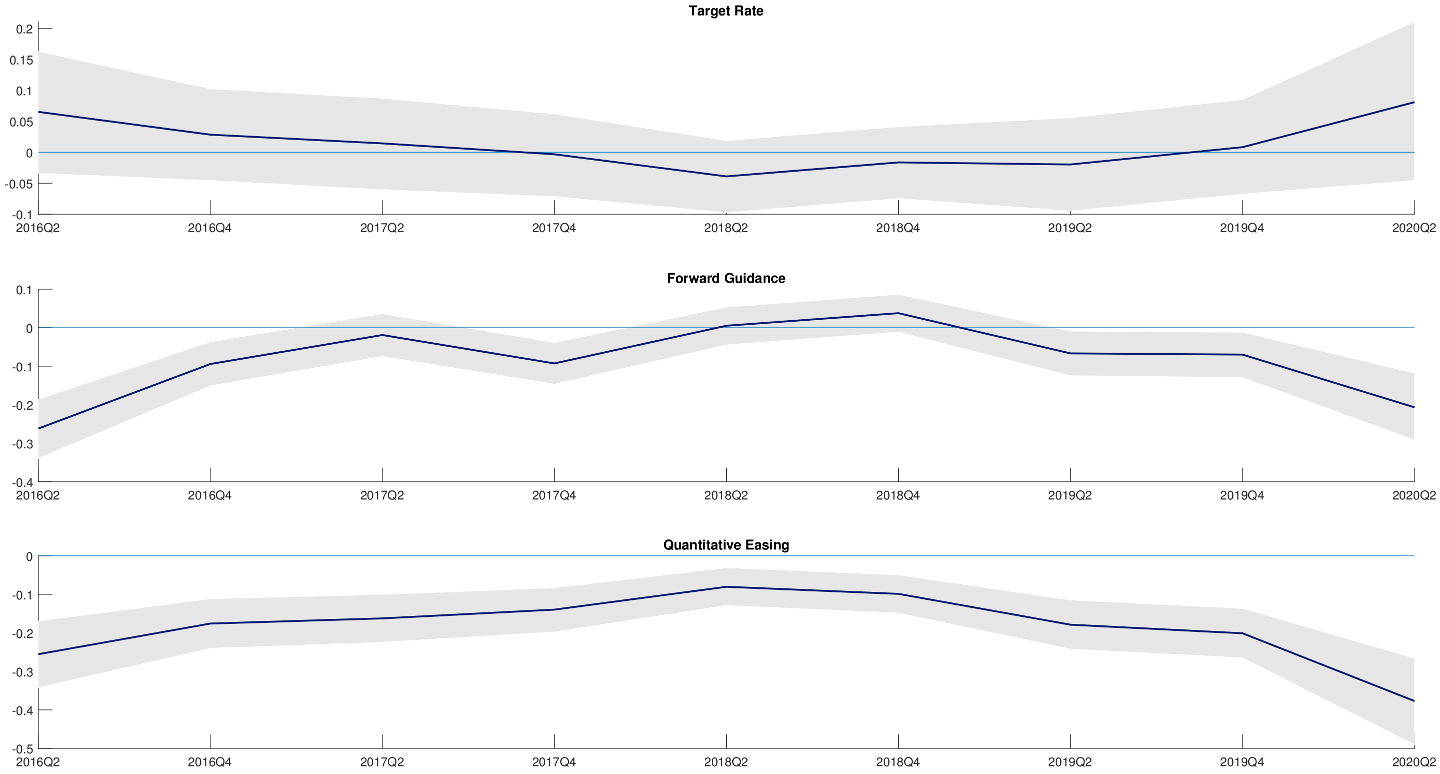
Notes: Impulse responses of country level inflation to a unit monetary policy shock in Italy, Lithuania, Luxembourg, Latvia, and Malta. Top row: Response to a Target Rate shock. Middle row: Response to a Forward Guidance shock. Bottom row: Response to a Quantitative Easing shock. The shaded bands denote the 68 percent confidence intervals.

Figure 4: Causal Effect Estimates - NLD to SVN



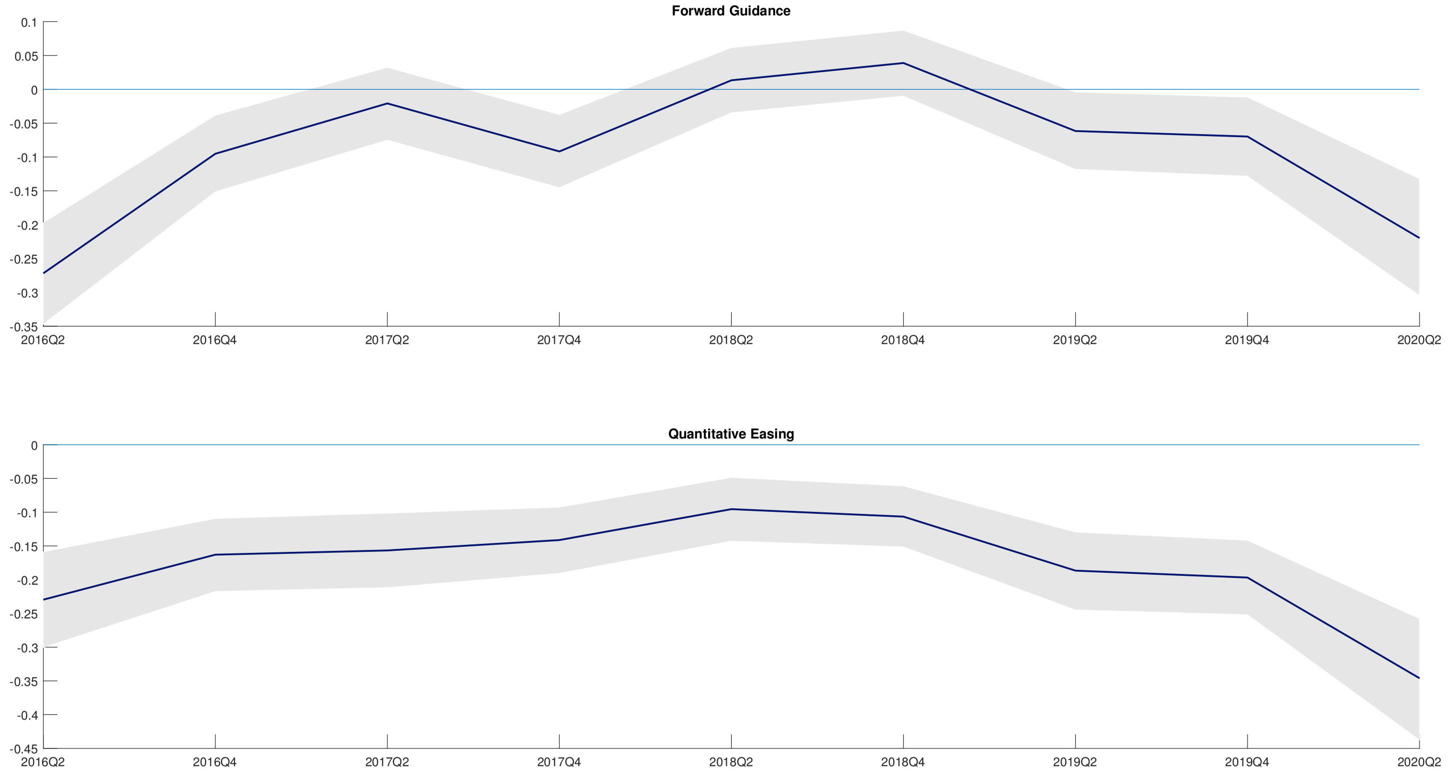
Notes: Impulse responses of country level inflation to a unit monetary policy shock in the Netherlands, Portugal, Slovakia and Slovenia. Top row: Response to a Target Rate shock. Middle row: Response to a Forward Guidance shock. Bottom row: Response to a Quantitative Easing shock. The shaded bands denote the 68 percent confidence intervals.

Figure 5: Aggregate OPP Results



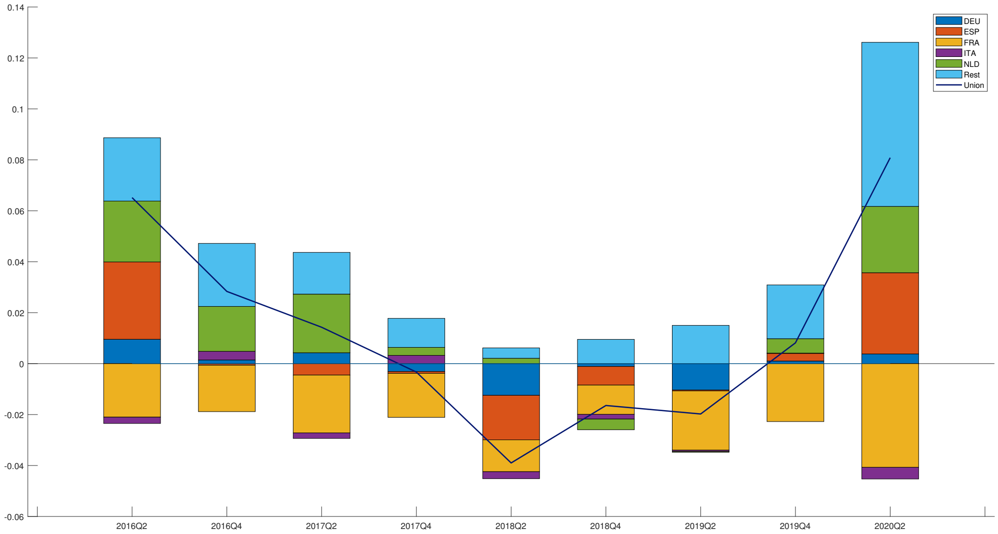
Notes: Top panel: Union level test statistic for the Target Rate instrument. Middle Panel: Union level test statistic for the Forward Guidance instrument. Bottom Panel: Union level test statistic for the Quantitative Easing instrument. Shaded areas represent impulse response and model uncertainty at 68 percent confidence.

Figure 6: Aggregate OPP Results Assuming ZLB



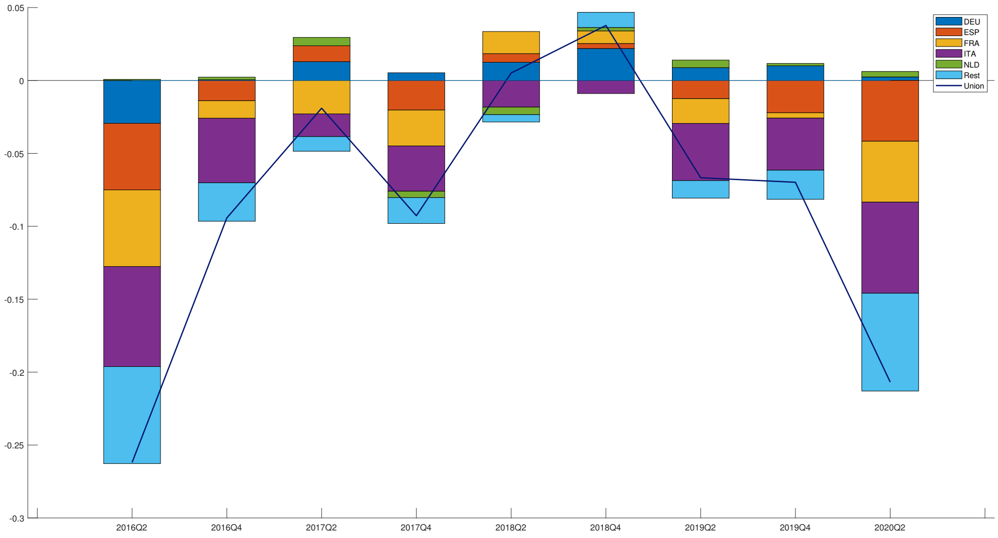
Notes: Top panel: Union level test statistic for the Forward Guidance instrument assuming the Zero Lower Bound is binding. Bottom Panel: Union level test statistic for the Quantitative Easing instrument assuming the Zero Lower Bound is binding. Shaded areas represent impulse response and model uncertainty at 68 percent confidence.

Figure 7: Country Level Contribution to Union - Target Rate



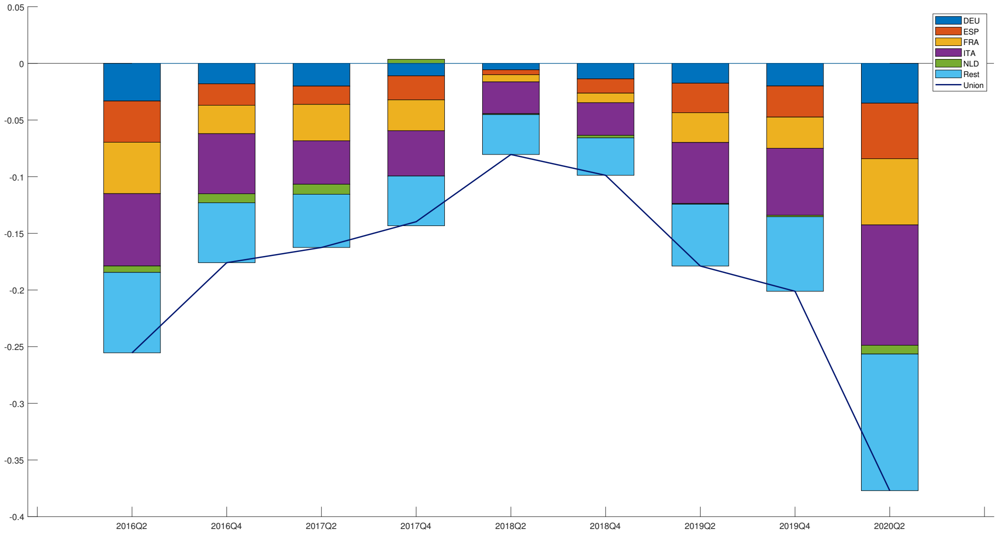
Notes: Contribution of member countries to the union level test statistic for the Target Rate instrument. For ease of legibility, the figure presents the contribution of the five largest economies, Germany, France, Italy, Spain, and the Netherlands, and the collective contribution of the rest of the member countries.

Figure 8: Country Level Contribution to Union - Forward Guidance



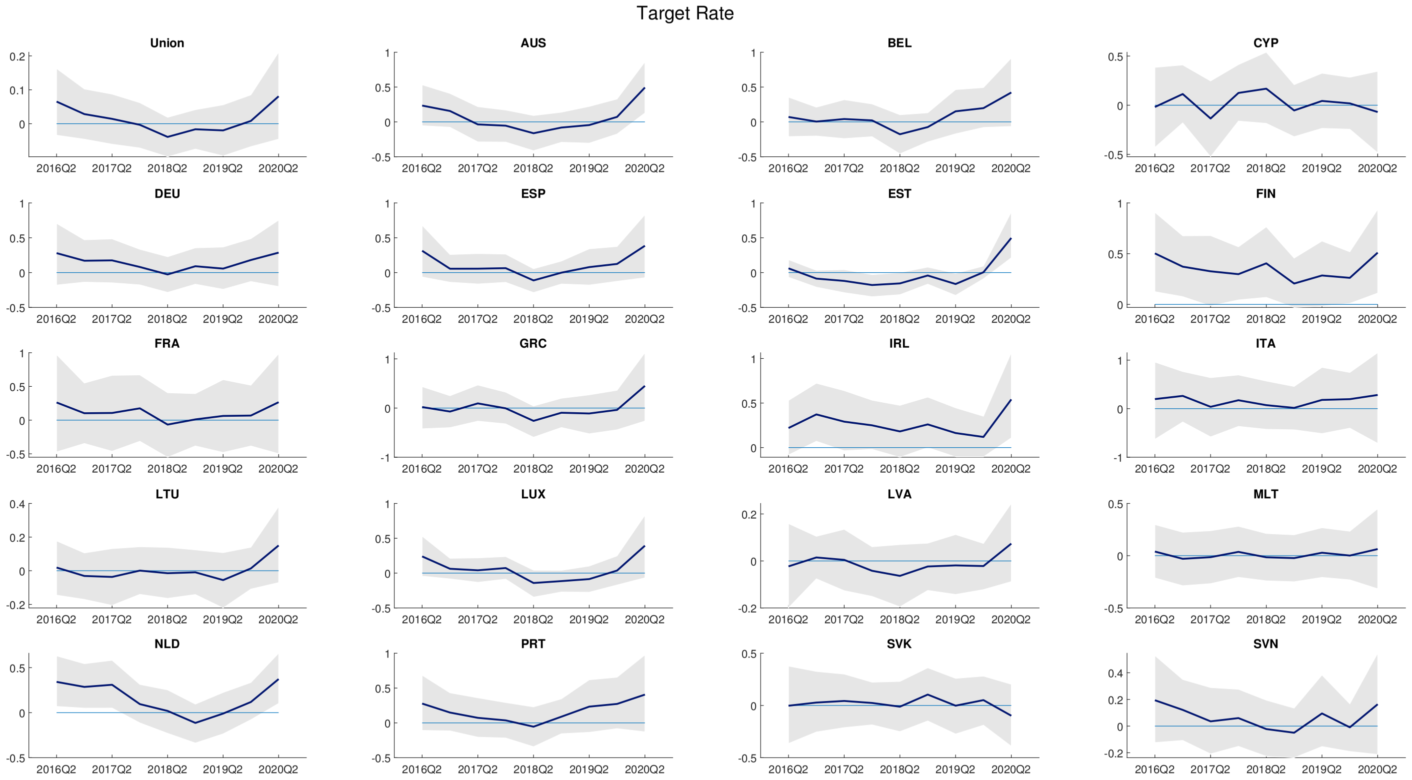
Notes: Contribution of member countries to the union level test statistic for the Forward Guidance instrument. For ease of legibility, the figure presents the contribution of the five largest economies, Germany, France, Italy, Spain, and the Netherlands, and the collective contribution of the rest of the member countries.

Figure 9: Country Level Contribution to Union - Quantitative Easing



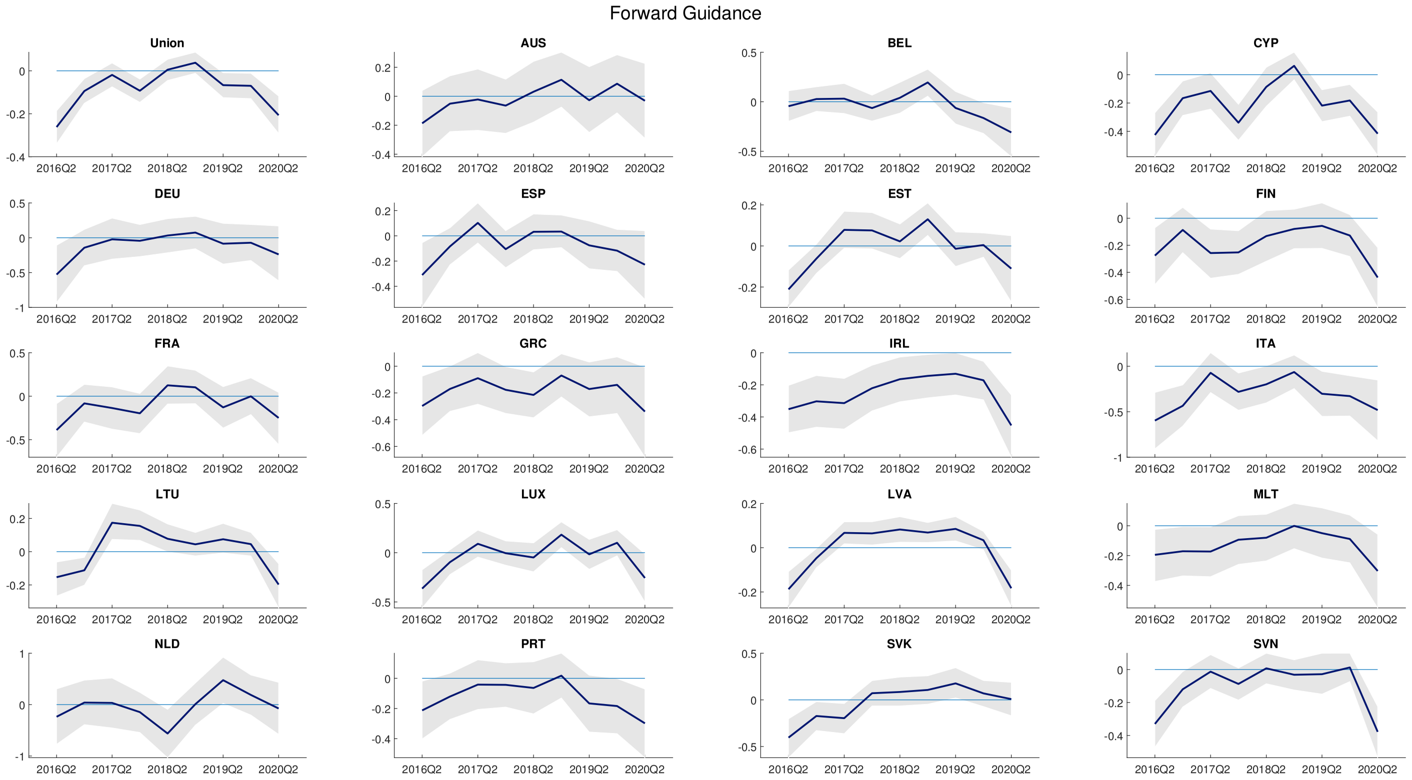
Notes: Contribution of member countries to the union level test statistic for the Quantitative Easing instrument. For ease of legibility, the figure presents the contribution of the five largest economies, Germany, France, Italy, Spain, and the Netherlands, and the collective contribution of the rest of the member countries.

Figure 10: Country Level OPP Results - Target Rate



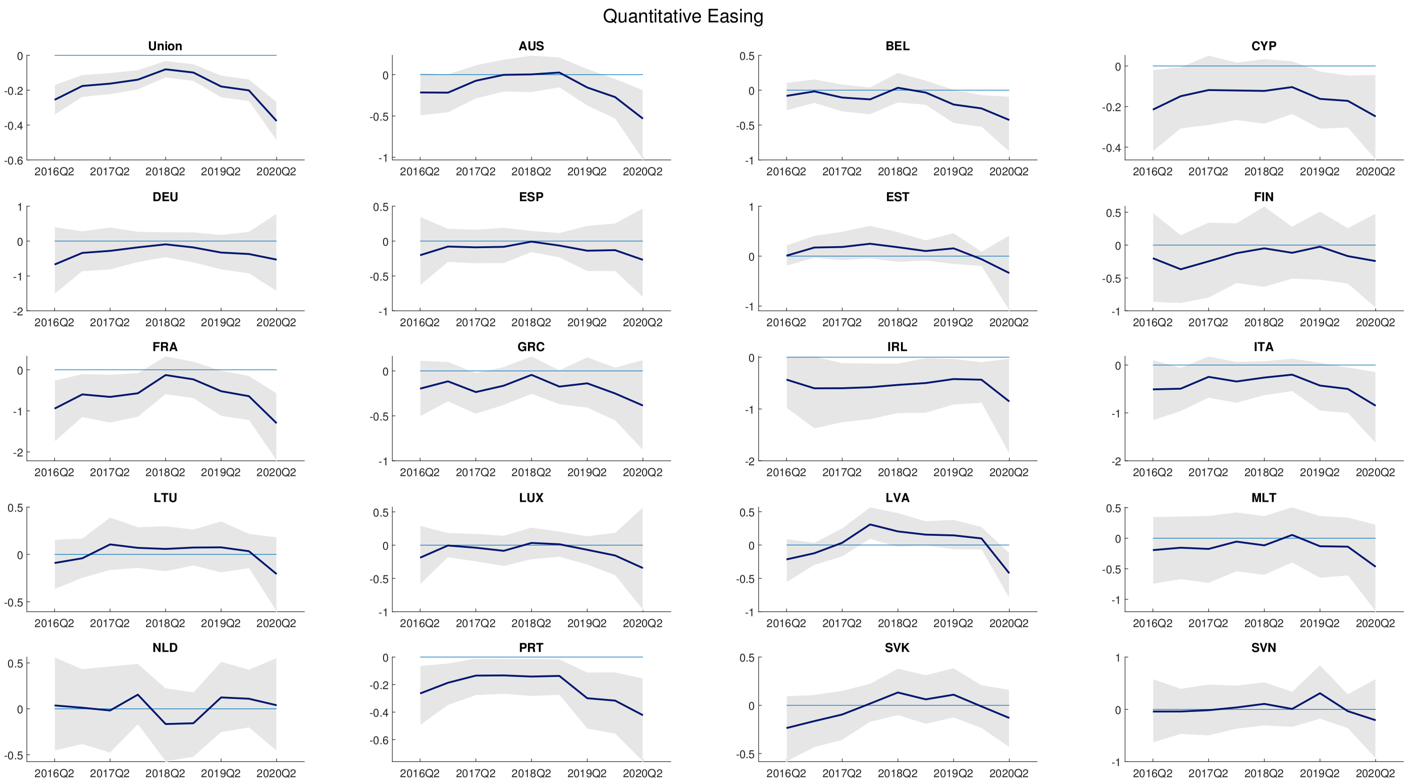
Notes: Union level test statistic for the Target Rate instrument and individual member country test statistcs for the Target Rate instrument. Shaded areas represent impulse response and model uncertainty at 68 percent confidence.

Figure 11: Country Level OPP Results - Forward Guidance



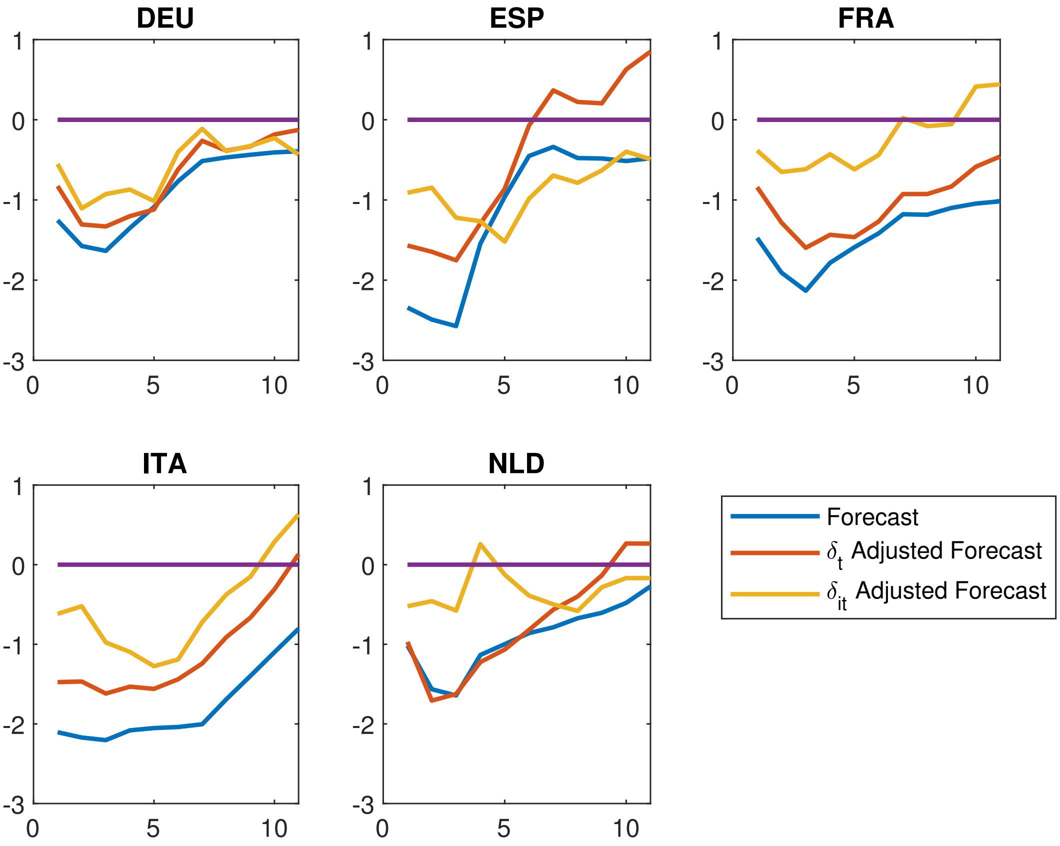
Notes: Union level test statistic for the Forward Guidance instrument and individual member country test statistcs for the Forward Guidance instrument. Shaded areas represent impulse response and model uncertainty at 68 percent confidence.

Figure 12: Country Level OPP Results - Quantitative Easing



Notes: Union level test statistic for the Quantitative Easing instrument and individual member country test statistcs for the Quantitative Easing instrument. Shaded areas represent impulse response and model uncertainty at 68 percent confidence.

Figure 13: Case Study: 2020Q2



Notes: Deviation from target of forecasted paths of inflation along with counterfactual paths for inflation if policy is adjusted by the union level statistic on the one hand and the country level statistic on the other.

Table 1: Rejections of Optimality

	2016Q2	2016Q4	2017Q2	2017Q4	2018Q2	2018Q4	2019Q2	2019Q4	2020Q2
ECB	F/Q	F/Q	Q	F/Q	Q	Q	F/Q	F/Q	F/Q
AUS								Q	T/Q
BEL						F		F/Q	F/Q
CYP	F/Q	F/Q		F			F/Q	F/Q	F/Q
DEU	F								
ESP	F								
EST	F			T	T	F	T		T
FIN	T/F	T	F	T/F	T			T	T/F
FRA	F/Q	Q	Q	Q			Q	Q	Q
GRC	F		Q	F	F				F
IRL	F	T/F	F/Q	F/Q	F/Q	T/F/Q	F/Q	F/Q	T/F/Q
ITA	F	F		F/Q			FG	F/Q	F/Q
LTU	F	F	F	F					F
LUX	F				F				
LVA	F	F	F	F/Q	F	F	F		F/Q
MLT	F	F	F						FG
NLD	T	T	T		F		F		T
PRT	F/Q	Q	Q	Q	Q	Q	Q	F/Q	F/Q
SVK	F	F	F				F		
SVN	F	F							F

Notes: The entries in the table denote instances where we can reject optimality of a policy instrument where T indicates that we can reject that the Target Rate was optimally set for that country, F indicates the same for the Forward Guidance instrument and Q the Quantitative Easing instrument.

Table 2: Relative Reduction in Country Specific Loss - 2020Q2

2020Q2	DEU	ESP	FRA	ITA	NLD
Adjust by δ_t	0.67	0.52	0.57	0.45	0.95
Adjust by δ_{it}	0.43	0.43	0.09	0.19	0.16

Notes: The first row shows the median expected loss after adjusting by the union level test statistic relative to the expected loss with no adjustment at the 2020Q2 decision. The second row shows the median expected loss after adjusting by the per country level test statistic relative to the expected loss with no adjustment at the 2020Q2 decision.

Table 3: Relative Reduction in Country Specific Loss

	AUS	BEL	CYP	DEU	ESP	EST	FIN	FRA	GRC	IRL
Adjust by δ_t	0.69	0.52	0.41	0.67	0.50	1.50	0.61	0.58	0.31	0.67
Adjust by δ_{it}	0.47	0.24	0.22	0.43	0.48	0.19	0.05	0.19	0.18	0.09
Ratio	0.68	0.45	0.52	0.65	0.96	0.12	0.08	0.32	0.59	0.13
	ITA	LTU	LUX	LVA	MLT	NLD	PRT	SVK	SVN	
Adjust by δ_t	0.51	1.57	0.74	2.11	0.49	0.98	0.33	1.94	0.94	
Adjust by δ_{it}	0.23	0.24	0.56	0.19	0.33	0.39	0.12	0.23	0.39	
Ratio	0.45	0.15	0.75	0.09	0.67	0.40	0.36	0.12	0.42	

Notes: The first two rows of each panel show the median expected loss after adjusting by the union level OPP statistic and the country level OPP statistic, respectively, relative to the expected loss with no adjustment. The bottom row show the ratio between the expected loss after adjusting according to the country level OPP statistic and the expected loss after adjusting according to the union level OPP.

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WORKING PAPER
2024 | 96